BILINEAR FORMS OVER QUATERNION ALGEBRAS

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▪ *ABSTRACT: In the present paper, it is proved that a quotient of the Witt ring of bilinear forms over $\mathcal{D}$ is isomorphic to Witt ring $W(F)$, where $\mathcal{D}$ is a division quaternion algebra over an 1-Hilbert field F of characteristic two.*

▪ *KEYWORDS: Witt ring; Witt-equivalence; quaternion algebra.*

1 Introduction

In the present paper, it is verified that some properties of bilinear forms over a skew field given in (Craven, 1982) and (Sladek, 1986) are valid over a skew field of characteristic two. In the sequel, when the skew field is one division quaternion algebra $\mathcal{D}$ over an 1-Hilbert field F of characteristic two, it is shown that the groups of square class $\mathcal{D}/S(\mathcal{D})$ and $\dot{F}/\dot{F}^2$ are isomorphic. This isomorphism induces an epimorphism of $W(\mathcal{D})$ into $W(F)$ by considering bilinear Witt-equivalence, according to Baeza and Moresi (1985) for characteristic two.

2 Preliminaries

For a skew field $\mathcal{D}$ of characteristic two, let $\dot{\mathcal{D}}$ denote the multiplicative group of all nonzero elements of $\mathcal{D}$, and let $S(\mathcal{D})$ denote the subgroup of $\mathcal{D}$ generated by $\dot{\mathcal{D}}^2 := \{s^2,\ s \in \dot{\mathcal{D}}\}$. In particular, since $aba^{-1}b^{-1} = a^2(a^{-1}b)^2(b^{-1})^2$, then $\dot{\mathcal{D}}^2$ contains the multiplicative commutators of $\mathcal{D}$.

The quotient group $\dot{\mathcal{D}}/S(\mathcal{D})$ is abelian (see Proposition 2.4(a) of (Craven, 1982)). The $\mathbb{Z}_2$ group ring of group $\dot{\mathcal{D}}/S(\mathcal{D})$ is denoted by $\mathbb{Z}_2[\dot{\mathcal{D}}/S(\mathcal{D})]$. Elements of group $\dot{\mathcal{D}}/S(\mathcal{D})$ are denoted by $\langle c\rangle$.

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We write $\langle c_1, \ldots, c_n \rangle$ for the element $\sum_{i=1}^{n} \langle c_i \rangle \in \mathbb{Z}_2[\dot{\mathcal{D}}/S(\mathcal{D})]$. Any element $\langle c_1, \ldots, c_n \rangle$ is said to be a form of dimension n over $\mathcal{D}$.

Definition 2.1. [Cf. (Baeza and Moresi, 1985), (Craven, 1982) and (Sladek, 1986)]. The Witt ring of $\mathcal{D}$ is ring $W(\mathcal{D}) := \left(\mathbb{Z}_2[\dot{\mathcal{D}}/S(\mathcal{D})]\right)/J$, where J is the ideal generated by all elements of the form

$$(1 + \langle c \rangle)(1 + \langle 1 + c \rangle) = \langle 1, c \rangle + \langle 1 + c, c(1 + c) \rangle,$$

for $c,\ 1 + c \in \dot{\mathcal{D}}$.

Two forms $\langle c_1, \ldots, c_n \rangle$ and $\langle d_1, \ldots, d_n \rangle$ are considered to be *equivalent* or *isometric*, and write $\langle c_1, \ldots, c_n \rangle \simeq \langle d_1, \ldots, d_n \rangle$ if they are equal to module J. We can also define the *determinant* map,

$$det : \mathbb{Z}_2\left[\dot{\mathcal{D}}/S(\mathcal{D})\right] \to \dot{\mathcal{D}}/S(\mathcal{D}),$$

by $det\langle c_1, \ldots, c_n \rangle = c_1 \cdots c_n S(\mathcal{D})$. Since every element of J has determinant equal to 1 module $S(\mathcal{D})$, the determinant is also well defined on elements of $W(\mathcal{D})$.

The following results have the same proofs as in the case of characteristic different from two.

Lemma 2.2. *[Lemma 4.2 of (Craven, 1982)]. For any* $c,\ d \in \dot{\mathcal{D}}$ *and any* $s_1,\ s_2 \in S(\mathcal{D})$ *such that* $cs_1 + ds_2 \neq 0$, *we have* $\langle c, d \rangle \simeq \langle cs_1 + ds_2, cd(cs_1 + ds_2) \rangle$.

Proposition 2.3. *[Proposition 4.4 of (Craven, 1982)] Assume we have* $\langle c_1, \ldots, c_n \rangle \in \mathbb{Z}_2[\dot{\mathcal{D}}/S(\mathcal{D})]$ *and* $z \in \dot{\mathcal{D}}$. *If there exist elements* $s_i \in S(\mathcal{D}) \cup \{0\}$ *such that* $z = \sum_{i=1}^{n} c_i s_i$, *then there exist* $d_2, \ldots, d_n \in \dot{\mathcal{D}}$ *such that* $\langle c_1, \ldots, c_n \rangle \simeq \langle z, d_2, \ldots, d_n \rangle$.

Proposition 2.4. *[Proposition 4.7 of (Craven, 1982)] If* $det\langle c_1, c_2 \rangle = det\langle d_1, d_2 \rangle$ *and* $\langle c_1, c_2 \rangle$ *and* $\langle d_1, d_2 \rangle$ *both represent some common element of* $\mathcal{D}$, *then* $\langle c_1, c_2 \rangle \simeq \langle d_1, d_2 \rangle$.

We observe that the chain equivalence for bilinear forms is also valid in the case of characteristic two. The proof is also the same as that of Lemma 4.10 and Remark 4.11(b) of (Craven, 1982).

Definition 2.5. Given the form $q = \langle c_1, \ldots, c_n \rangle$ over $\mathcal{D}$ we define

$$D_{\mathcal{D}}q := \{c_1 s_1 + \cdots + c_n s_n \in \dot{\mathcal{D}} :\ s_1, \ldots, s_n \in S(\mathcal{D}) \cup \{0\}\}$$

and

$$\tilde{D}_{\mathcal{D}}q := \{d \in \dot{\mathcal{D}} :\ q \simeq \langle d, d_2, \ldots, d_n \rangle, \text{for some } b_2, \ldots, b_n \in \dot{\mathcal{D}}\}.$$

Proposition 2.3 implies that $D_{\mathcal{D}}q \subseteq \tilde{D}_{\mathcal{D}}q$. By using the determinant of form $\langle 1, d\rangle$, we can prove that if $c \in \tilde{D}_{\mathcal{D}}\langle 1, d\rangle$ then $\langle 1, d\rangle \simeq \langle c, cd\rangle$. Thus, we have the following results as in the case of characteristic different from two, whose proofs follow in an analogous way.

Theorem 2.6. *[Theorem 1.4 of (Sladek, 1986)] If $d \in \dot{\mathcal{D}}$ then $\tilde{D}_{\mathcal{D}}\langle 1, d\rangle$ is the subgroup of $\dot{\mathcal{D}}$ generated by set $D_{\mathcal{D}}\langle 1, d\rangle$.*

We note that if $\mathcal{D}$ is a field, then $\tilde{D}_{\mathcal{D}}\langle 1, d\rangle = D_{\mathcal{D}}\langle 1, d\rangle$.

The following proposition is also immediate.

Proposition 2.7. *[Proposition 1.5 of (Sladek, 1986)] Let $c, d, c', d' \in \dot{\mathcal{D}}$. Then*

(i) $d \in \tilde{D}_{\mathcal{D}}\langle 1, c\rangle$ *iff* $c \in \tilde{D}_{\mathcal{D}}\langle 1, d\rangle$.

(ii) $\tilde{D}_{\mathcal{D}}\langle 1, c\rangle \cap \tilde{D}_{\mathcal{D}}\langle 1, d\rangle \subseteq \tilde{D}_{\mathcal{D}}\langle 1, cd\rangle$.

(iii) $\langle c, d\rangle \simeq \langle c', d'\rangle$ *iff* $abS(\mathcal{D}) = c'd'S(\mathcal{D})$ *and* $\tilde{D}_{\mathcal{D}}\langle c, d\rangle = \tilde{D}_{\mathcal{D}}\langle c', d'\rangle$.

3 Group $\dot{\mathcal{D}}/S(\mathcal{D})$ for a quaternion algebra $\mathcal{D}$.

In this section, we assume that $\mathcal{D} = \left(\frac{a, b}{F}\right]$ is a quaternion algebra over F, where F is a field of characteristic two. We denote the standard basis of $\mathcal{D}$ by $\{1, i, j, k\}$. Thus $i^2 = a,\ j^2 = j + b,\ k = ij$ and $ij + ji = i$. For $x = \alpha + \beta j + \gamma i + \delta k \in \mathcal{D}$, the conjugate of x is element $\bar{x} := \alpha + \beta + \beta j + \gamma i + \delta k$. The usual trace $T : \mathcal{D} \to F$ and norm $N : \dot{\mathcal{D}} \to F$ maps are given by $T(x) = x + \bar{x}$ and $N(x) = x.\bar{x}$. Thus, every element $x \in \mathcal{D}$ satisfies equation $x^2 + T(x)x + N(x) = 0$. We have that $N(\dot{\mathcal{D}}) = D\big([1, b] \perp \langle a\rangle[1, b]\big)$, where $D\big([1, b] \perp \langle a\rangle[1, b]\big) := \{\alpha^2 + \alpha\beta + b\beta^2 + a(\gamma^2 + \gamma\delta + b\delta^2) \in \dot{F},\ \alpha, \beta, \gamma, \delta \in F\}$ is the set of elements represented by Pfister form $[1, b]\perp\langle a\rangle[1, b]$. The quadratic form $[1, b]\perp\langle a\rangle[1, b]$ is also represented by $\langle\langle a, b]]$, and algebra $\mathcal{D}$ is a skew field if and only if $\langle\langle a, b]]$ is not hyperbolic. We also have that $\langle\langle a, b]]$ is hyperbolic or anisotropic, (See Chapter IV, Corollary 3.2 of (Baeza, 1978)). Let $\mathcal{D}_0$ be the following subset of $\mathcal{D}$, $\mathcal{D}_0 := \{x \in \mathcal{D} \mid x^2 \in F\}$. We have $\mathcal{D}_0 = F \oplus Fi \oplus Fk = \{x \in \mathcal{D} \mid T(x) = 0\}$, and $N_{|\mathcal{D}_0} \simeq [1] \perp \langle a\rangle[1, b]$, (See §1 of (Baeza, 1981)). The equation above implies that $\mathcal{D}_0^2 = N(\mathcal{D}_0)$. We assume that $\mathcal{D}$ is a skew field and we define the group homomorphism $\tilde{N} : \dot{\mathcal{D}}/S(\mathcal{D}) \longrightarrow \dot{F}/\dot{F}^2$ by $\tilde{N}(dS(\mathcal{D})) = N(d)\dot{F}^2$.

Lemma 3.1. *Let $\mathcal{D} = \left(\frac{a, b}{F}\right]$ be a division quaternion algebra over F. Then* $D\langle\langle a, b]] = \big(D([1]\perp\langle a\rangle[1, b])\big)\big(D([1]\perp\langle a\rangle[1, b])\big) = \dot{\mathcal{D}}_0^2.\dot{\mathcal{D}}_0^2$.

Proof: Since $D\langle\langle a, b]]$ is a subgroup of $\dot{F}$ and $[1]\perp\langle a\rangle[1, b]$ is a subform of $\langle\langle a, b]]$, inclusion $\supseteq$ is clear. Otherwise, given $x = m^2 + mn + bn^2 + a(u^2 + uv + bv^2)$ we desire to get $\alpha, \beta, \gamma, \alpha_1, \beta_1, \gamma_1 \in F$ such that

$$x = \big(\alpha^2 + a(\beta^2 + \beta\gamma + b\gamma^2)\big)\big(\alpha_1^2 + a(\beta_1^2 + \beta_1\gamma_1 + b\gamma_1^2)\big).$$

If $n = v = 0$ then $x = m^2 + an^2 \in \big(D([1]\perp\langle a\rangle[1,b])\big)^2$.

If $n \neq 0$ or $v \neq 0$ then $n^2 + av^2 \neq 0$, because $n^2 + av^2 \in D\langle\langle a,b]]$ and $\langle\langle a,b]]$ is anisotropic. In this case, by straightforward calculations, we obtain solution: $\alpha = v$, $\beta = a^{-1}n$, $\gamma = 0$, $\alpha_1 = a(mv + un)(n^2 + av^2)^{-1}$, $\beta_1 = 1, \gamma_1 = (auv + mn)(n^2 + av^2)^{-1}$. Thus,

$\big([1]\perp\langle a\rangle[1,b]\big)(\alpha,\beta,\gamma).\big([1]\perp\langle a\rangle[1,b]\big)(\alpha_1,\beta_1,\gamma_1)$ is equal to x. ∎

Theorem 3.2. *Let* $\mathcal{D} = (\frac{a,b}{F}]$ *be a skew field. Then the sequence of groups*

$$1 \longrightarrow N(\dot{\mathcal{D}}) \hookrightarrow \dot{F} \longrightarrow \frac{\dot{\mathcal{D}}}{S(\mathcal{D})} \longrightarrow \frac{N(\dot{\mathcal{D}})}{\dot{F}^2} \longrightarrow 1$$

is exact.

Proof: Using Lemma 3.1, the proof is the same as that of the Theorem 2.1 of (Sladek, 1986). ∎

A field F is an 1-Hilbert field if there exists only one anisotropic 2-fold Pfister quadratic form $\langle\langle c,d]]$, up to isometric (see Theorem 2.2 of (Santos, 1997)). Therefore $\langle\langle c,d]]$ is universal, that is, $D\langle\langle c,d]] = \dot{F}$ Thus we have

Corollary 3.3. *Let* $\mathcal{D} = \left(\frac{a,b}{F}\right]$ *be a skew field, where F is 1-Hilbert field. Then* $\tilde{N} : \dot{\mathcal{D}}/S(\mathcal{D}) \longrightarrow \dot{F}/\dot{F}^2$ *is an isomorphism.*

4 Ring $W(\mathcal{D})$ for a division quaternion algebra $\mathcal{D}$

Now we proceed to consider $I^n(\mathcal{D})$, the ideal of $W(\mathcal{D})$ generated by the subset of forms $\{\langle 1,c_1\rangle.\cdots.\langle 1,c_n\rangle,\ c_i \in \dot{\mathcal{D}},\ i = 1,2,\ldots,n\}$. We denote $I^1(\mathcal{D})$ by $I(\mathcal{D})$. Let Det be the restriction of determinant map to $I(\mathcal{D})$. Similarly to the commutative case, we have

Lemma 4.1. *The map:* $Det : I(\mathcal{D}) \longrightarrow \frac{\dot{\mathcal{D}}}{S(\mathcal{D})}$ *defined by* $Det(\sum\langle c_i,d_i\rangle) = \prod c_i d_i.S(\mathcal{D})$ *induce an isomorphism*

$$dis : \frac{I(\mathcal{D})}{I^2(\mathcal{D})} \xrightarrow{\sim} \frac{\dot{\mathcal{D}}}{S(\mathcal{D})}.$$

Proof: It is clear that $Det : I(\mathcal{D}) \longrightarrow \dot{\mathcal{D}}/S(\mathcal{D})$ is an epimorphism, and since $Det\big(\sum_i a_i\langle 1,c_i\rangle\langle 1,d_i\rangle\big) = 1$, we have $I^2(\mathcal{D}) \subseteq Ker(Det)$. Conversely suppose that $q = \langle c_1,\ldots,c_n\rangle \in I(\mathcal{D})$ and $Det(q) = 1$. From $\langle c_1,c_2\rangle \equiv \langle 1,c_1c_2\rangle(mod.I^2(\mathcal{D}))$ we get $\langle c_1,c_2,c_3,c_4\rangle \equiv \langle 1,c_1c_2c_3c_4\rangle(mod.I^2(\mathcal{D}))$. As $dim(q)$ is even, it follows that $q \in I^2(\mathcal{D})$ or $q \in \langle c,d\rangle + I^2(\mathcal{D})$, for some $\langle c,d\rangle \in W(\mathcal{D})$. If $q = \langle c,d\rangle + q_1,\ q_1 \in I^2(\mathcal{D})$ then $1 = Det(q) = cd$ (since $Det(q_1) = 1$), and so $d = c$ in $\dot{\mathcal{D}}/S(\mathcal{D})$. Thus $q = 2\langle c\rangle + q_1 = q_1 \in I^2(\mathcal{D})$, and $Ker(Det) \subseteq I^2(\mathcal{D})$. ∎

Lemma 4.2. *The natural isomorphism $dis : I(\mathcal{D})/I^2(\mathcal{D}) \xrightarrow{\sim} \dot{\mathcal{D}}/S(\mathcal{D})$ induces an homomorphism $\bar{N}_1 : I(\mathcal{D})/I^2(\mathcal{D}) \longrightarrow I(F)/I^2(F)$ such that the following diagram*

$$\begin{array}{ccc} \dfrac{I(\mathcal{D})}{I^2(\mathcal{D})} & \xrightarrow{\bar{N}_1} & \dfrac{I(F)}{I^2(F)} \\ dis \downarrow & & \downarrow dis \\ \dfrac{\dot{\mathcal{D}}}{S(\mathcal{D})} & \xrightarrow{\tilde{N}} & \dfrac{\dot{F}}{\dot{F}^2} \end{array}$$

commutes. In particular, if F is an 1-Hilbert field then $\bar{N}_1$ is an isomorphism.

Proof: Since every element of $I(\mathcal{D})/I^2(\mathcal{D})$ is the form $\langle c,d\rangle + I^2(\mathcal{D})$, we define $\bar{N}_1 : I(\mathcal{D})/I^2(\mathcal{D}) \longrightarrow I(F)/I^2(F)$ by putting $\bar{N}_1(\langle c,d\rangle + I^2(\mathcal{D})) := dis^{-1}\big(\tilde{N}\big(dis(\langle c,d\rangle + I^2(\mathcal{D}))\big)\big)$. Since dis is an isomorphism, we have $\bar{N}_1 : I(\mathcal{D})/I^2(\mathcal{D}) \longrightarrow I(F)/I^2(F)$ is well defined, and it is clear that the diagram above commutes. If F is an 1-Hilbert field, from Corollary 3.3 $\bar{N}_1$ is an isomorphism.

From the commutative diagram above we have $dis\big(\bar{N}_1\big(\langle c,d\rangle + I^2(\mathcal{D})\big)\big) = \tilde{N}\big(dis\big(\langle c,d\rangle + I^2(\mathcal{D})\big)\big) = \tilde{N}\big(cS(\mathcal{D})\big)\tilde{N}\big(dS(\mathcal{D})\big) = N(c).N(d).\dot{F}^2 = dis\big(\langle N(c),N(d)\rangle + I^2(F)\big)$. Thus we have $\bar{N}_1\big(\langle c,d\rangle + I^2(\mathcal{D})\big) = \langle N(c),N(d)\rangle + I^2(F)$. In particular

$$\bar{N}_1\big(\langle 1,c\rangle + I^2(\mathcal{D})\big) = \langle 1,N(c)\rangle + I^2(F). \tag{1}$$

Now we define $\varphi : I(\mathcal{D}) \longrightarrow I(F)/I^2(F)$ by putting $\varphi\big(\sum\langle c_i,d_i\rangle\big) = \sum\big(\langle N(c_i),N(d_i)\rangle + I^2(\mathcal{D})\big)$. The above relation implies for any $c,\ d \in \dot{\mathcal{D}}$

$$\varphi(\langle 1,c\rangle).\varphi(\langle 1,d\rangle) \equiv \langle 1,N(c)\rangle.\langle 1,N(d)\rangle\big(mod.I^3(F)\big). \tag{2}$$

So $\varphi(\langle 1,c\rangle.\langle 1,d\rangle) = \varphi(\langle 1,c,d,cd\rangle) = \langle 1,N(c),N(d),N(c).N(d)\rangle + I^2(F) = \big(\langle 1,N(c)\rangle + I^2(F)\big).\big(\langle 1,N(d)\rangle + I^2(F)\big) = \varphi(\langle 1,c\rangle).\varphi(\langle 1,d\rangle)$. Thus

$$\varphi(\langle 1,c\rangle.\langle 1,d\rangle) \equiv \langle 1,N(c)\rangle.\langle 1,N(d)\rangle\big(mod.I^3(F)\big). \tag{3}$$

Lemma 4.3. *Let $\mathcal{D} = \left(\dfrac{a,b}{F}\right]$ be a division quaternion algebra over the field F, and $c \in \dot{\mathcal{D}}\backslash S(\mathcal{D})$. Then $N\big(\tilde{D}_{\mathcal{D}}\langle 1,c\rangle\big) \subseteq D_F\langle 1,N(c)\rangle$.*

Proof: Let d be in $\tilde{D}_{\mathcal{D}}\langle 1,c\rangle$. If $d \in S(\mathcal{D})$, then $N(d) \in D_F\langle 1,N(c)\rangle$. If $d \notin S(\mathcal{D})$, then $\langle 1,d\rangle \neq 0$ in $W(\mathcal{D})$ and by hypothesis $\langle 1,c\rangle \simeq \langle d\rangle\langle 1,c\rangle$. It follows that $\langle 1,c\rangle.\langle 1,d\rangle = 0$ in $W(\mathcal{D})$. By relation 3 we deduce

$$\langle 1,N(c)\rangle.\langle 1,N(d)\rangle \in I^3(F).$$

Now, from Hauptsatz of Arason and Pfister page 173 of (Arason and Pfister, 1971), we deduce

$$\langle 1,N(c)\rangle.\langle 1,N(d)\rangle = 0$$

in $W(F)$. This relation implies that $N(d) \in D_F\langle 1, N(c)\rangle$, since $\langle 1, N(c)\rangle \neq 0$ in $W(F)$. ∎

Theorem 4.4. *Let* $\mathcal{D} = \left(\frac{a,b}{F}\right]$ *be a division quaternion algebra over the 1-Hilbert field* F*. Then, there exists an epimorphism* $\phi : W(\mathcal{D}) \to W(F)$ *such that* $W(\mathcal{D})/\phi^{-1}(J(F))$ *is isomorphic to* $W(F)$.

Proof: The isomorphism of Corollary 3.3 extends to an isomorphism $\tilde{N}_1 : \mathbb{Z}_2[\dot{\mathcal{D}}/S(\mathcal{D})] \xrightarrow{\sim} \mathbb{Z}_2[\dot{F}/\dot{F}^2]$ given by $\tilde{N}_1(\sum\langle c_i\rangle) = \sum\langle N(c_i)\rangle$. From now on, it is suffices to show that $\tilde{N}_1(J(\mathcal{D})) \subseteq J(F)$.

For $c \in \dot{\mathcal{D}}$ such that $1 + c \in \dot{\mathcal{D}}$ by definition $\tilde{N}_1(\langle 1, c, 1+c, c(1+c)\rangle) = \langle 1, N(c), N(1+c), N(c).N(1+c)\rangle$. As $1 + c \in \tilde{D}_{\mathcal{D}}\langle 1, c\rangle$, from Lemma 4.3 we have $N(1+c) = z^2 + N(c)t^2$, $z, t \in F$. If $z = 0$ or $t = 0$ then $\tilde{N}_1(\langle 1, c, 1+c, c(1+c)\rangle = 0 \in J(F)$. If $z,\ t \in \dot{F}$, then $\tilde{N}_1(\langle 1, c, 1+c, c(1+c)\rangle) = \langle 1, N(c), z^2 + N(c)t^2, N(c).(z^2 + N(c)t^2)\rangle$. Since $z \neq 0$, $\quad r = (t/z) \in \dot{F}$, and so we write $\tilde{N}_1(\langle 1, c, 1+c, c(1+c)\rangle) = \langle 1, N(c)r^2, 1 + N(c)r^2, N(c)r^2.(1 + N(c)r^2)\rangle \in J(F)$, and so $N(J(\mathcal{D})) \subseteq J(F)$. ∎

Conclusions

The author do not know if the Hauptsatz of Arason and Pfister is valid over Witt ring of skew field. We conjecture that, if q is a positive-dimensional anisotropic form over $\mathcal{D}$, q belongs to $I^n\mathcal{D}$, then the dimension of q is at least 2^n. We then would obtain the reciprocal of 3:

$$\psi(\langle 1, x\rangle.\langle 1, y\rangle) \equiv \langle 1, N^1(x)\rangle.\langle 1, N^{-1}(y)\rangle\big(mod.I^3(\mathcal{D})\big), \tag{4}$$

where $x,\ y \in \dot{F}$ and $\psi : I(F) \to I(\mathcal{D})/I^2(\mathcal{D})$. This relation would imply that $N(\tilde{D}_{\mathcal{D}}\langle 1, c\rangle) = D_F\langle 1, N(c)\rangle$ in Lemma 4.3, which would furnish isomorphism $W(\mathcal{D}) \simeq W(F)$ in Theorem 4.4.

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- RESUMO: Neste artigo é provado que um quociente do anel de Witt das formas bilineares sobre $\mathcal{D}$ é isomorfo ao anel de Witt $W(F)$, onde $\mathcal{D}$ é uma álgebra de quatérnios com divisão sobre o corpo 1-Hilbertiano F de característica dois.

- PALAVRAS-CHAVE: Anel de Witt; Witt-equivalência; álgebra de quatérnios.

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