BIVARIATE STOCHASTIC VOLATILITY MODELS APPLIED TO AIR POLLUTION DATA

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▪ *ABSTRACT: In this paper, we introduce the use of bivariate stochastic volatility models applied to air pollution data. Recent introduced stochastic volatility models used to analyze financial time series are considered to estimate the volatilities of the weekly ozone measures considering two different regions of Mexico city in the period from January 01, 1990 to December 31, 2005. The Bayesian analysis is developed using Markov Chain Monte Carlo (MCMC) methods to simulate samples for the joint posterior distribution of interest.*

▪ *KEYWORDS: Stochastic volatility models; air pollution data; ozone pollution; Bayesian analysis; MCMC methods.*

1 Introduction

Air pollution is one of the most important pollution problems in the world. Among many existing pollutants, ozone is one of the mainly pollutants affecting a large number of cities. In many cities, as for example, Mexico city, environmental authorities have implemented measures aiming to reduce the level of pollutants. When ozone concentration stay high levels for a given period of time, individuals exposed to the pollutant may experience serious health problems (see for example, Bell et al, 2004; Loomis et al, 1996; Wilson et al, 1980).

Several statistical methods have been used to predict the violation of an air quality standard (see for example, Horowitz, 1980; Roberts, 1979a, 1979b; Smith, 1989; Achcar et al, 2008a; Javits, 1980; Leadbetter, 1991; Raftery, 1989).

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Among the different statistical models used to model air pollution data, we could consider time series modeling of the daily or weekly average pollution data (see for example, Loomis et al, 1996; or Achcar et al, 2008b).

In this way, we could use stochastic volatility models (SV) (see for example, Ghysels et al, 1996; Kim et al, 1998; Meyer and Yu, 200). This family of models have been extensively used to analyze financial time series (see for example, Danielsson, 1994; Yu, 2002), as a powerful alternative for the usual existing ARCH type models (autoregressive conditional heteroscedastic) introduced by Engle (1982) and the generalized autoregressive conditional heteroscedastic (GARCH) models introduced by Bollerslev (1986).

The use of SV-type models has many advantages to analyze time series since they assume two processes to model the series: a process to model the observations and a process to model the latent volatilities.

Bayesian inference approach using Markov Chain Monte Carlo (MCMC) methods (see for example, Gelfand and Smith, 1990; Smith and Roberts, 1993) have been considered to analyze SV models since we can have great difficulties using standard classical inference approach, as high dimensionality, likelihood function with no closed form and high computational cost.

In this paper, we consider the use of SV models to analyze a data set collected from the monitoring network of Mexico city (www.sma.df.gov.mx/simat/) corresponding to 15 years (from January 01, 1990 to December, 31, 2005) of the weekly average ozone measurements for two different regions of Mexico city.

2 Bivariate stochastic volatility models

Different classes of multivariate stochastic volatility models are introduced in the literature (see for example, Yu and Meyer, 2005).

Let $\mathbf{y_t} = (y_{1t}, y_{2t})', t = 1, ..., N$ be two time series where y_{1t} and y_{2t} usually are the logarithms of the returns centralized around their averages, with model,

$$\mathbf{y} = H_t \boldsymbol{\epsilon_t} \tag{1}$$

where $\boldsymbol{\epsilon_t} = (\epsilon_{1t}, \epsilon_{2t})'$ is the vector of error components for the two series and H_t is a 2 x 2 matrix given by

$$H_t = diag\left(e^{h_{1t}/2}, e^{h_{2t}/2}\right) \tag{2}$$

That is,

$$\begin{aligned} y_{1t} &= e^{h_{1t}/2}\epsilon_{1t} \\ y_{2t} &= e^{h_{2t}/2}\epsilon_{2t} \end{aligned} \tag{3}$$

for $t = 1, 2, ..., N$.

Let us assume that $\boldsymbol{\epsilon_t} = (\epsilon_{1t}, \epsilon_{2t})$ has a bivariate normal distribution,

$$\boldsymbol{\epsilon}_{\mathbf{t}} \sim N\left(\mathbf{0}, \Sigma_{\epsilon}\right) \tag{4}$$

where the variance - covariance matrix is given by,

$$\Sigma_{\epsilon} = \begin{pmatrix} 1 & \rho_{\epsilon} \\ \rho_{\epsilon} & 1 \end{pmatrix} \tag{5}$$

Also assume that $\mathbf{h_t} = \left(h_{1,t}, h_{2,t}\right), t = 1, 2, ..., N$ is a vector of latent variables defined by autoregressive $AR\left(1\right)$ models,

$$\begin{aligned} h_{1,1} &= \mu_1 + \eta_{11} \\ h_{2,1} &= \mu_2 + \eta_{21} \\ h_{1,t} &= \mu_1 + \phi_{11}\left(h_{1,t-1} - \mu_1\right) + \eta_{1t} \\ h_{2,t} &= \mu_2 + \phi_{22}\left(h_{2,t-1} - \mu_2\right) + \eta_{2t} \end{aligned} \tag{6}$$

for $t = 2, 3, ..., N; 0 < \phi_{11} < 1; 0 < \phi_{22} < 1;$
$\boldsymbol{\eta}_{\mathbf{t}} = \left(\eta_{1t}, \eta_{2t}\right)$ is assumed to have a bivariate normal distribution given by,

$$\boldsymbol{\eta}_{\mathbf{t}} \sim N\left\{\mathbf{0}; diag\left(\sigma^2_{\eta_1}; \sigma^2_{\eta_2}\right)\right\} \tag{7}$$

where $\mathbf{0} = (0, 0)'$; and $diag\left(\sigma^2_{\eta_1}; \sigma^2_{\eta_2}\right)$ is a 2 x 2 diagonal matrix.

From (1), we observe that $E\left(\mathbf{y_t}\right) = \mathbf{0}$ and the variance - covariance matrix for $\mathbf{y_t}$ is given by,

$$var\left(\mathbf{y_t}\right) = H_t'\Sigma_{\epsilon}H_t = \begin{pmatrix} e^{h_{1t}} & \rho_{\epsilon}e^{h_{1t}/2}e^{h_{2t}/2} \\ \rho_{\epsilon}e^{h_{1t}/2}e^{h_{2t}/2} & e^{h_{2t}} \end{pmatrix} \tag{8}$$

Thus, $\mathbf{y_t} = \left(y_{1t}, y_{2t}\right)'$ has a bivariate normal distribution with density

$$\begin{aligned} f\left(y_{1t}, y_{2t} \mid h_{1,t}, h_{2,t}\right) &= \frac{1}{2\pi\sqrt{\left(1-\rho^2_{\epsilon}\right)e^{h_{1t}+h_{2t}}}} \times \\ &\times \exp\left\{-\frac{1}{2\left(1-\rho^2_{\epsilon}\right)}\left[\frac{y^2_{1t}}{e^{h_{1t}}} + \frac{y^2_{2t}}{e^{h_{2t}}} - \frac{2\rho_{\epsilon}y_{1t}y_{2t}}{e^{h_{1t}/2}e^{h_{2t}/2}}\right]\right\} \end{aligned} \tag{9}$$

Observe that functions of $h_{1,t}$ and $h_{2,t}$ gives the volatility of the time series.

Also observe that from (6) and (7), the latent variables $\mathbf{h_t} = \left(h_{1,t}, h_{2,t}\right)$ have normal distributions,

$$\begin{aligned} h_{l1} &\sim N\left(\mu_l; \sigma^2_{\eta_l}\right) \\ h_{lt} &\mid h_{l,t-1} \sim N\left\{\mu_l + \phi_{ll}\left(h_{l,t-1} - \mu_l\right); \sigma^2_{\eta_l}\right\} \end{aligned} \tag{10}$$

for $l = 1, 2; t = 2, ..., N$.

A special case of this model is given when we assume that the error components are independent, that is, $\rho_\epsilon = 0$. Let us denote this model as "model 1", in the case of $\rho_\epsilon = 0$ and as "model 2" when ρ_ϵ is unknown.

3 A Bayesian analysis

Assuming a constant correlation for the error components model defined by (1) ... (6), denoted as "model 2", the likelihood function is given by,

$$
\begin{aligned}
\prod_{t=1}^{N} p\left(\mathbf{y_t} \mid \mathbf{h_t}\right) \quad \propto \quad & \left(1-\rho_\epsilon^2\right)^{-N/2} \times \\
& \times \exp\left\{-\frac{1}{2}\left[\sum_{t=1}^{N} h_{1t} + \sum_{t=1}^{N} h_{2t}\right]\right\} \times \\
& \times \exp\left\{-\frac{1}{2\left(1-\rho_\epsilon^2\right)}\left[\sum_{t=1}^{N} y_{1t}^2 e^{-h_{1t}} + \sum_{t=1}^{N} y_{2t}^2 e^{-h_{2t}} - \right.\right. \\
& \left.\left. -2\rho_\epsilon \sum_{t=1}^{N} y_{1t} y_{2t} e^{-h_{1t}/2} e^{-h_{2t}/2}\right]\right\}
\end{aligned} \tag{11}
$$

Assuming $\rho_\epsilon^2 = 0$, we have "model 1", that is, independent error components.

For a Bayesian analysis of "model 2" (see for example Bernardo and Smith, 1995), let us assume the following prior distributions for the parameters $\phi_{11}, \phi_{22}, \sigma_{\eta_1}^2, \sigma_{\eta_2}^2, \mu_1, \mu_2$ and ρ_ϵ :

$$
\begin{aligned}
\phi_{ll} &\sim beta\left(a_{ll}, b_{ll}\right); l = 1, 2; \\
\sigma_{\eta_l}^2 &\sim IG\left(c_l, d_l\right); l = 1, 2; \\
\mu_l &\sim N\left(0; e_l^2\right); l = 1, 2; \\
\rho_\epsilon &\sim U\left[-1, 1\right],
\end{aligned} \tag{12}
$$

where all hyperparameters for the prior distributions are assumed to be known; *beta* (a, b) denotes a beta distribution with:

$$\text{mean } a/\left(a+b\right) \text{ and variance } ab/\left[\left(a+b\right)^2\left(a+b+1\right)\right];$$

$IG\left(c, d\right)$ denotes a inverse gamma distribution with:

$$\text{mean } d/\left(c-1\right) \text{ and variance } d^2/\left[\left(c-1\right)^2\left(c-2\right)\right], c > 2;$$

And $U\left(-1, 1\right)$ denotes a uniform distribution defined in the interval $\left(-1, 1\right)$. We further assume a prior independence among the parameters.

With the latent variables $h_{1,t}$ and $h_{2,t}, t = 1, ..., N$ defined by (6), the joint posterior distribution for $\boldsymbol{\varphi_1} = (\boldsymbol{\theta_1}, \mathbf{h})$, where $\boldsymbol{\theta_1} = \left(\mu_1, \mu_2, \phi_{11}, \phi_{22}, \sigma^2_{\eta_1}, \sigma^2_{\eta_2}, \rho_\epsilon\right)$, is given by,

$$\pi\left(\boldsymbol{\varphi_1} \mid \mathbf{y}\right) \propto \pi\left(\boldsymbol{\theta_1}\right) g\left(\mathbf{h_1}\right) \prod_{t=2}^{N} g\left(\mathbf{h_t} \mid \mathbf{h_{t-1}}, \boldsymbol{\theta_1}\right) \times \times \prod_{t=1}^{N} p\left(\mathbf{y_t} \mid \mathbf{h_t}\right) \tag{13}$$

where $\pi\left(\boldsymbol{\theta_1}\right)$ is the joint prior distribution defined by (12); $g\left(\mathbf{h_1}\right)$ is the joint density for $\mathbf{h_1} = \left(h_{1,1}, h_{2,1}\right)$ (see 10) given by,

$$g\left(\mathbf{h_1}\right) \propto \prod_{l=1}^{2} \left(\sigma^2_{\eta_l}\right)^{-1/2} \exp\left[-\frac{1}{2\sigma^2_{\eta_l}} \left(h_{l1} - \mu_l\right)^2\right],$$

$g\left(\mathbf{h_t} \mid \mathbf{h_{t-1}}, \boldsymbol{\theta_1}\right)$ is the joint density for $\mathbf{h_t} = \left(h_{1t}, h_{2t}\right), t = 2, 3, ..., N$ (see 10) given by,

$$g\left(\mathbf{h_t} \mid \mathbf{h_{t-1}}, \boldsymbol{\theta_1}\right) \propto \prod_{l=1}^{2} \prod_{t=2}^{N} \left(\sigma^2_{\eta_l}\right)^{-1/2} \times \times \exp\left\{-\frac{1}{2\sigma^2_{\eta_l}} \left[h_{lt} - \mu_l - \phi_{ll}\left(h_{l,t-1} - \mu_l\right)\right]^2\right\},$$

and $\prod_{t=1}^{N} p\left(\mathbf{y_t} \mid \mathbf{h_t}\right)$ is the likelihood function defined in (11).

Samples of the joint posterior distribution (13) are generated using MCMC methods as the Gibbs Sampling algorithm or the Metropolis - Hastings algorithm (see for example, Gelfand and Smith, 1990; Smith and Roberts, 1993).

A great simplification in the simulation of the samples from (13) is obtained using the software Winbugs (Spiegelhalter et al, 2003), where we only need to specify the distribution for the data and the prior distribution for the parameters.

4 Model selection

Different Bayesian discrimination methods are introduced in the literature to choose the best BSV model to be fitted by the data.

The Bayesian information criterion (BIC) introduced by Schwarz (1978), assume a penalized log - likelihood function given by,

$$BIC = \ln L\left(\widehat{\boldsymbol{\theta}}\right) - \frac{1}{2} p \ln\left(N\right) \tag{14}$$

where $L\left(\widehat{\boldsymbol{\theta}}\right)$ is the maximized likelihood function at the maximum likelihood estimator; p is the dimension of the parameter vector and N is the sample size.

Carlin and Louis (2000) consider a modification of the BIC replacing $\ln L\left(\widehat{\boldsymbol{\theta}}\right)$ by $E\left[\ln L\left(\widehat{\boldsymbol{\theta}}\right)\right]$ assuming the posterior distribution for θ. Larger values of BIC indicates better models.

The deviance information criterion (DIC) (see Spiegelhalter et al, 2002) is given by,

$$DIC = \widehat{D} + 2p_D \tag{15}$$

where $\widehat{D}$ is the deviance evaluated in the posterior mean and p_D is the effective number of parameters in the model, given by $p_D = \overline{D} - \widehat{D}$, where $\overline{D}$ is the posterior mean deviance.

Smaller values of DIC indicates the best BSV models, these values also could be negatives.

5 An application to the weekly ozone averages in Mexico city

In this section, we apply the stochastic volatility modeling to the case of weekly ozone averages in the Metropolitan Area of Mexico city from January 01, 1990 to December 31, 2005. As a special case we consider the time series data corresponding to two regions of the Metropolitan Area of Mexico city: The Northeast (NE) and the Center (CE) regions.

In Figure 1, we have the plots of the weekly ozone averages versus weeks for the period from January 01, 1990 to December 31, 2005. From Figure 1, we observe a decreasing trend for the weekly ozone averages for these two regions of Mexico city, especially after the 400^{th} week (close to the year 1998).

In Figure 2, we have the plots of the log - returns $y_{jt} = \log\left(z_{jt}/z_{j,t-1}\right)$ centralized on their averages where z_{jt} is the weekly ozone average in time t for region j, where j=1 (region NE) and j=2 (region CE).

Assuming "model 1", that is, the basic stochastic volatility model defined by equations (1), (2), (4), (5) and (6) assuming $\rho_\epsilon = 0$ (independent time series) and prior distributions (12), that is,$\phi_{ll} \sim beta\,(1,1)\,;\tau_{\eta_l} = 1/\sigma^2_{\eta_l} \sim Gamma\,(1,1)$ and $\mu_l \sim N\,(0;100)$ for $l = 1,2$, we have in Table 1, the posterior summaries of interest obtained using the software Winbugs (Spiegelhalter et al, 2003). In the simulation of samples from the joint posterior distribution (13), we considered a "burn - in - sample" of size 5000 to eliminate the effects of the initial values for the Gibbs sampling algorithm; after this "burn - in - sample" period, we selected 1000 Gibbs samples by taking every 10^{th} sample to have approximately uncorrelated samples.

In Figure 3, we have the Monte Carlo estimates for the square roots of the volatilities for the regions NE and CE of Mexico city based on the 1000 final simulated Gibbs samples and assuming "model 1".

Table 1 - Posterior summaries

model	parameter	mean	S.D.	95% credible interval
model 1	τ_{η_1}	4.8270	1.3870	(2.8580; 8.1280)
DIC = -128.708	τ_{η_2}	3.9570	1.0880	(2.3930; 6.4800)
$\rho_D = 192.244$	μ_1	-3.0700	0.0633	(-3.1900; -2.9510)
	μ_2	-2.9860	0.0763	(-3.1300; -2.8380)
	ϕ_{11}	0.4945	0.1491	(0.1840; 0.7526)
	ϕ_{22}	0.6334	0.0855	(0.4595; 0.7791)
model 2	τ_{η_1}	9.4260	2.3890	(5.3840; 14.6500)
DIC = -745.865	τ_{η_2}	7.4360	1.8600	(4.6590; 11.6800)
$\rho_D = 186.903$	μ_1	-3.0220	0.0700	(-3.1630; -2.8880)
	μ_2	-2.9160	0.0744	(-3.0740; -2.7720)
	ϕ_{11}	0.7307	0.0818	(0.5350; 0.8580)
	ϕ_{22}	0.6985	0.0807	(0.5263; 0.8335)
	ρ_ϵ	0.7546	0.0165	(0.7209; 0.7849)

From the plots of Figure 3, we observe that for both regions NE and CE there is a more stable volatility after the 400^{th} week (close to the year 1998) and most important, a smaller value for the volatility of the ozone weekly average.

From these results, we see that the implemented measures considered by the environmental authorities of Mexico city has been efficient for the decreasing and stabilization of the ozone pollution.

Similarly, using the same Gibbs sampling scheme described for "model 1", we have in Table 1, the posterior summaries for the parameters of "model 2" introduced in section 2.

For "model 2", a stochastic volatility model with constant correlation for the error components, we used prior information considering the results of "model 1" assumed in the first step of the Bayesian analysis: $beta\,(1,1)$ priors for ϕ_{ll}; $gamma\,(5,1)$ prior for $\tau_{\eta_1} = 1/\sigma^2_{\eta_1}$; $gamma\,(4,1)$ prior for $\tau_{\eta_2} = 1/\sigma^2_{\eta_2}$; normal $N\,(-3;100)$ priors for $\mu_l, l = 1,2$ and an uniform $U\,(-1,1)$ prior for ρ_ϵ. Observe that we are using a kind of empirical Bayes approach (see for example, Carlin and Louis, 2000).

In Figure 4, we have the Monte Carlo estimates for the square roots of the volatilities assuming "model 2".

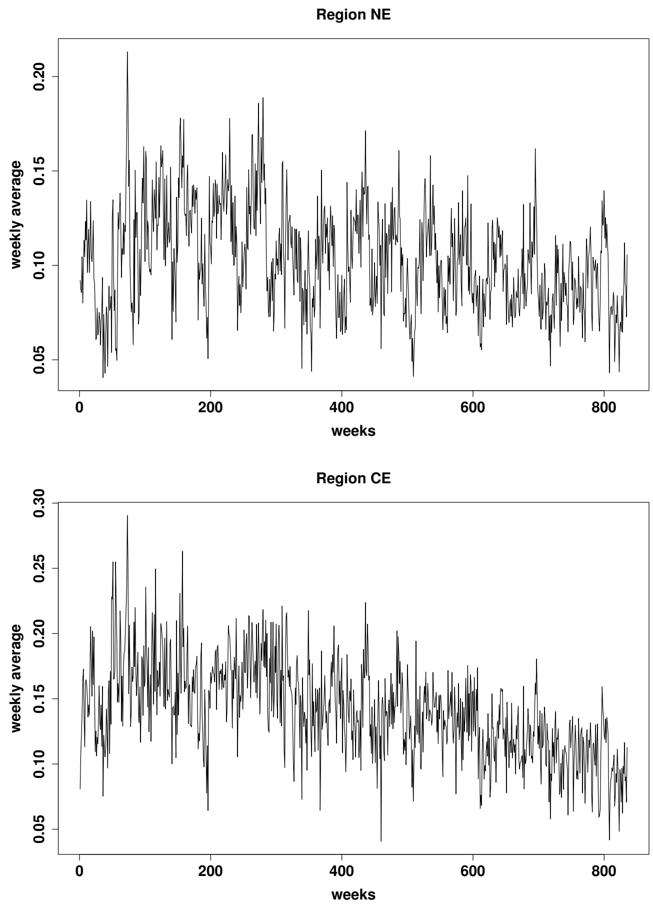

Figure 1 - Weekly ozone averages for the regions NE and CE for the period January 01, 1990 to December 31, 2005.

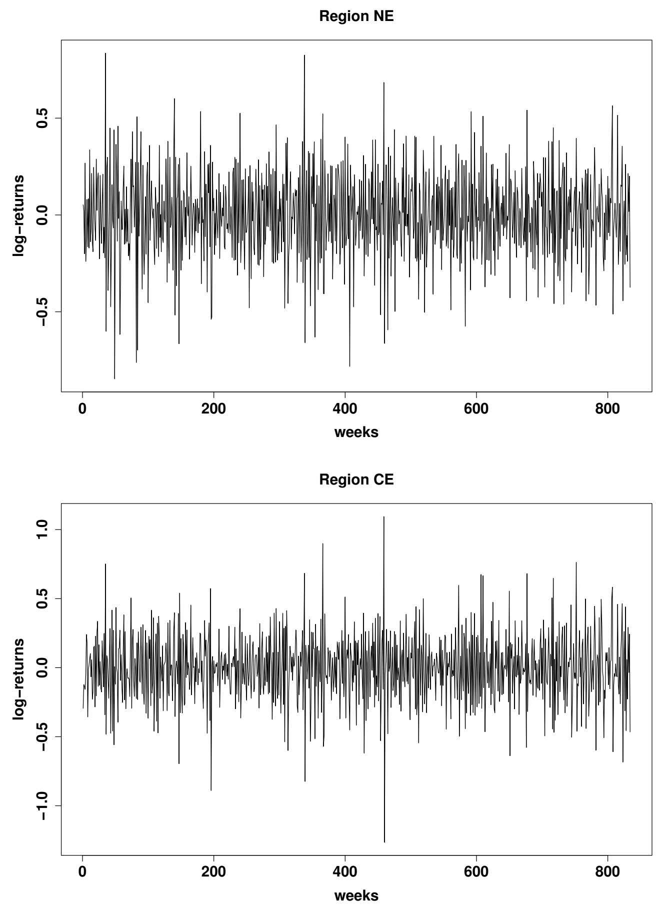

Figure 2 - Log-returns centralized in the averages for regions NE and CE.

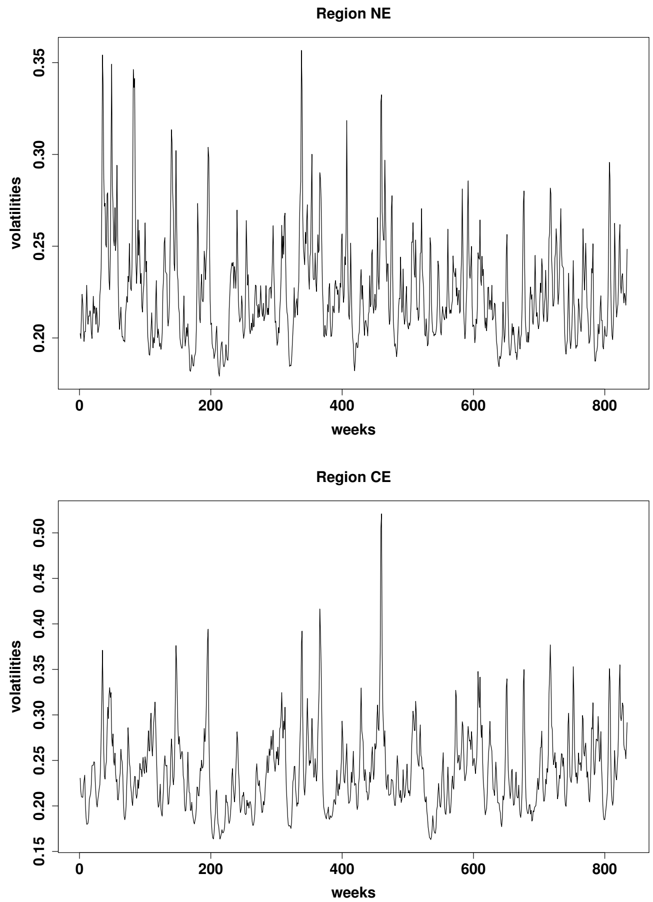

Figure 3 - Square roots of the volatilities for regions NE and CE ("model 1").

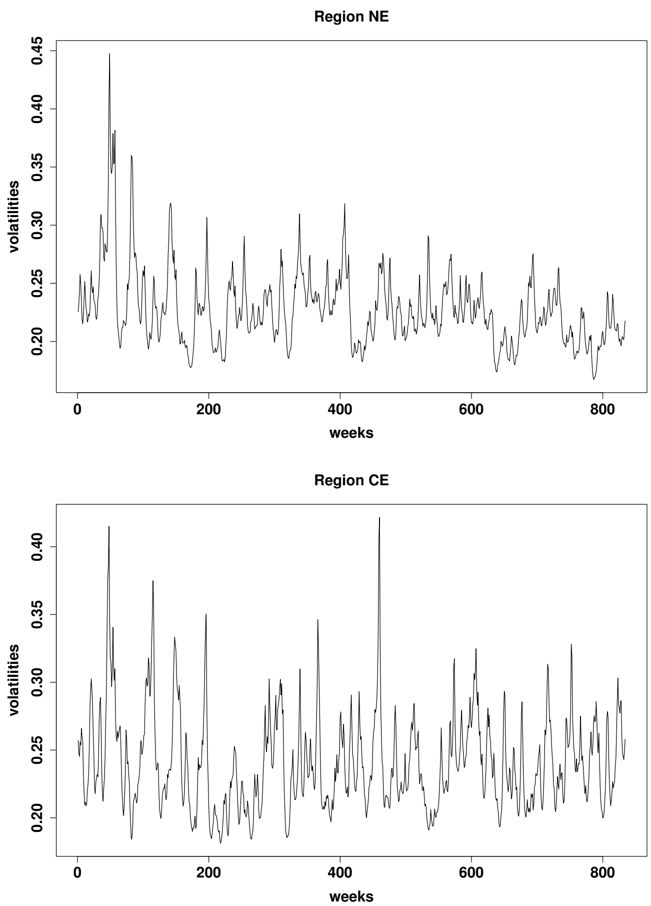

Figure 4 - Square roots of the volatilities for regions NE and CE (”model 2”).

In Table 1, we also have the Monte Carlo estimates for DIC (see section 4) considering the two models. From the results of table 1, we observe that "model 2" (error components dependent) is the best model to be fitted by the ozone pollution data of Mexico city for the regions NE and CE (smaller value of DIC given by -745.865).

It is important to observe from Figure 4 assuming "model 2" (the best fitted model), that for both regions NE and CE, there is smaller and more stable volatility for the ozone weekly measures after the 400^{th} week (close to the year 1998).

Conclusions

Note that even though the volatility decreases in all cases, we have a more stable plot, with rare exceptions, after week 600. This week corresponds roughly to the end of year 2001 and beginning of year 2002. It is interesting to call attention to the fact that in the year 2000 we have the culmination of a series of measures that had been taken by environmental authorities of Mexico since 1990 aiming to reduce the levels of ozone in big cities in the country and in particular in Mexico City. The series of measures may be roughly described as follows. In 1990, cars circulating in the Metropolitan Area of Mexico City would have to undergo periodic inspection of their mechanical condition. Additionally, restrictions were imposed on the circulation of car according to the ending number of their license plates. In 1997, further restrictions were implemented in terms of allowing clean cars to circulate freely. Additionally, in 1999, manufacturers were encouraged to produce cars with cleaner and modern technology. The production of such cars was made compulsory in 2001. Therefore, we may see that some measures taken by the environmental authorities have produced a positive effect towards the aim of decreasing and stabilizing the level of ozone measurements in Mexico City and in modifying its long term behavior. Finally, we would like to point out that besides their advantages when compared to the usual ARCH and GARCH models (mentioned earlier in this paper), stochastic volatility models also have an advantage when compared to non-homogeneous Poisson models (see Achcar et al., 2007). The advantage is that besides indicating on average, the number of peaks that occur in a given period of time, stochastic volatility models also permit to analyze if the variability of the measurements stay stable. That is important since environmental authorities are interested not only in decreasing pollution levels but also in keeping the measurements stabilized in a low level.

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- RESUMO: Neste artigo, nós introduzimos o uso de modelos de volatilidade estocástica bivariados aplicados em dados de poluição do ar. Modelos de volatilidade estocástica recentemente introduzidos usados para analisar séries financeiras ao longo do tempo são considerados para estimar as volatilidades das medidas semanais de ozônio considerando duas regiões diferentes da cidade do México no período de 01 de janeiro de 1990 a 31 de dezembro de 2005. A análise Bayesiana é desenvolvida utilizando os métodos de Monte Carlo Cadeias de Markov (MCMC) para simular amostras da distribuição conjunta a posterióri de interesse.

- PALAVRAS-CHAVE: Modelos de volatilidade estocástica; dados de poluição do ar; poluição por ozônio; análise Bayesiana; métodos MCMC.

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