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ARTICLE

The limits of conformity analysis under the Newcomb-Benford law and the COVID-19 pandemic in Brazil[1]

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Abstract

This research investigated the possible divergences in results due to the use of the Newcomb-Benford Law to assess the conformity of information regarding infections and deaths caused by the SARS-CoV-2 coronavirus, from transversal clippings made two years after the arrival of the COVID-19 pandemic in Brazil, through the comparative analysis of the results of statistics related to the Z test, the Chi-square test (X^2), the Kolmogorov-Smirnov test (KS), and also the mean absolute deviation (MAD). For this, two distinct samples were analyzed: one referring to data accumulated over two years of the pandemic in Brazil, from 02/25/2020 to 02/25/2022; and another exclusively to data from the day the pandemic was completed two years of its arrival in Brazil, therefore exclusively on 02/25/2022. Thus, it was possible to observe that the Brazilian epidemiological information system, governed by the health policies adopted by the Government, did not function properly during the COVID-19 pandemic, and still, there was no improvement in quality, even after two years of the pandemic. Additionally, evidence was observed that the data series relating to the contaminations and deaths caused by the SARS-CoV-2 coronavirus present behavior distinct from the expected frequency distributions according to the Newcomb-Benford Law, either as a result of a natural process or due to particular issues of the Brazilian information system itself, for example, logistics, data distortions, forms of geographic aggregation, errors and/or negligence, among other factors.

Keywords: Null hypothesis; Information; Health; Applied quantitative methods.

1. Introduction

Contrary to other epidemic events, the COVID-19 pandemic, caused by the SARS-CoV-2 coronavirus, received special attention from the academic community, the press, and governments around the world in real-time. As a result, the amount and availability of data provided a unique information base for studies and actions toward scientific evidence-based interventions (Silva; Figueiredo Filho, 2021).

Among so many studies, several kinds of research were conducted to analyze and validate the data on the amounts of contamination and deaths resulting from the pandemic of COVID-19 in Brazil and the world, and one of the tools used for this was the Newcomb-Benford Law (Newcomb, 1881; Benford, 1938).

The Newcomb-Benford Law, hereafter referred to as the NB Law, considers certain sets of data representing the occurrence of events of various natures in real-life or mathematical tables. The occurrences of the first and second digits of the numbers totaling these occurrences have a higher probability of being initiated by digits of lower value, obeying a particular logarithmic distribution (Lee; Han; Jeong, 2020).

In epidemic and/or pandemic contexts, the NB Law could be used to gauge the accuracy of health information systems (Gómez-Camponovo *et al*., 2016; Manrique-Hernández; Fernández-Niño; Idrovo, 2017), with special attention to cases of infection and death caused by the SARS-CoV-2 coronavirus (Idrovo; Manrique-Hernández, 2020; Kennedy; Yam, 2020; Lee; Han; Jeong, 2020; Manrique-Hernández *et al*., 2020; Carmo; Caneppele; Nunes, 2021; Galvêas; Barros Jr.; Fuzo , 2021; Silva; Figueiredo Filho, 2021; Adam; Tsarsitalidou, 2022).

In the case of Brazil, a particular situation was observed because, besides the difficulties imposed by the pandemic itself, in June 2020, the Brazilian government interrupted the flow of information regarding the quantities of infections and deaths by COVID-19 by claiming that it would change the way this information was shared (Carmo; Caneppele; Nunes, 2021; Idrovo; Manrique-Hernández; Nino, 2021). This further drew the attention of researchers to the need to assess the quality of information about the pandemic in the country. After all, the reliability of this type of information is essential for epidemiological studies and for the proposition of public health strategies, both aimed at tackling this disease that has cost the lives of almost 700,000 Brazilians (692,461 until 12/21/2022-16:57) (Brasil, 2022b).

Despite the results of the studies focused on the assessment of the informational quality of health systems during the pandemic of COVID-19, Druică, Oancea, and Vâlsan (2018) make some reservations about the use of the NB Law for compliance assessment, both the deconstruction of data sets evaluated, as well as concerning its application to longitudinal data series, compared to cross-sectional clippings, and also, concerning the statistical tests used to evaluate the adjustment of data series to the NB Law, as well as, concerning the influence of the number of observations integrating the analyzed samples.

In this context, by taking as a basis the results of studies of a related nature, and also, as a starting point, the results of the studies of Druică, Oancea, and Vâlsan (2018) and Carmo, Caneppele, and Nunes (2021), the present research aimed to analyze possible divergences in the results arising from the application of the NB Law for the evaluation of conformity of information regarding the cases of contamination and deaths caused by the SARS-CoV-2 coronavirus. Especially from a transversal clipping performed two years after the arrival of the COVID-19 pandemic in Brazil, through the comparative analysis of the results of statistics related to null hypothesis significance tests (NHST), that is, the Z test, the Chi-square test (X^2) and the Kolmogorov-Smirnov test (KS), and also the mean absolute deviation (MAD), as an evaluative metric of conformity based on the NB Law.

Although it takes as a basis the results of the research conducted by Druică, Oancea, and Vâlsan (2018) and Carmo, Caneppele, and Nunes (2021), this scientific investigation is justified by the possibility of presenting an unprecedented and relevant analytical contribution about the application of the NB Law in the process of conformity assessment of information regarding COVID-19 data in Brazil.

In this sense, it is worth noting that this research differs from the study of Druică, Oancea, and Vâlsan (2018) since it questioned the applicability of tests Z, X^2, KS, and MAD, among others, in the adjustment process to the NB Law, but from exclusively financial data. Similarly, the difference concerning the research conducted by Carmo, Caneppele, and Nunes (2021) is because, even using the Z and MAD tests to evaluate the adjustment of COVID-19 data in Brazil to the NB Law, this research was based only on state-level observations regarding the cases of contamination and deaths related to the first semester of the pandemic. In addition, this study used data series in a longitudinal clipping, while the present research used data series in a transversal clipping, and also analyzed data from the municipal perspective.

1.1 Theoretical background

Although recurrently used as a support tool in anomaly and fraud detection, NB's Law has been applied to the process of analyzing data series of diverse natures, among them: genome data; unstable nuclei half-lives; toxic emissions data; tax auditing; accounting; election data; stock markets; birth data; rivers; first letter words; elementary particle decay rates; astrophysical measurements, among others (Lee; Han; Jeong, 2020).

The NB Law, first proposed by Newcomb (1881), and later extended by Benford (1938), states that first and second digit totalizers of a series of occurrences of real-life events or mathematical tables tend to follow the frequency distribution described in Table 1.

Table 1. Frequency of in 1st and 2nd digits according to NB Law

Digit	1^{th} place	2^{nd} place
1	0.30103	0.11389
2	0.17609	0.10882
3	0.12494	0.10433
4	0.09691	0.10433
5	0.07918	0.09668
6	0.06695	0.09337
7	0.05799	0.09035
8	0.05115	0.08757
9	0.04576	0.08500
0	-	0.11968

Source: elaborated by the authors from Druică, Oancea, and Vâlsan (2018), Lee, Han, and Jeong (2020), Silva and Figueiredo Filho (2021).

The frequency distributions shown in Table 1 can be obtained from the logarithmic formulation described by Equation 1, proposed by Benford (1938), in which a represents the digit for which you want to calculate the expected relative frequency according to NB's Law.

$$F_a = log\left(\frac{a+1}{a}\right) \quad (1)$$

Since the frequency distribution obtained from the NB Law behaves like an exponential distribution, it may be used in analyzing the behavior of infectious diseases that evolve over time (Silva; Fiqueiredo Filho, 2021), such as the COVID-19 pandemic caused by the SARS-CoV-2 coronavirus.

Because of unprecedented pack of precedent, speed, and extent of damage, the COVID-19 pandemic required information on global proportions and raised concerns about the validity of the numbers of deaths and cases reported by governments worldwide (Kennedy; Yam, 2020).

Specifically in Brazil, the national and international concerns were heightened due to the government's denial of the pandemic that led to changes in the way it shared information. Additionally, the Brazilian ministry of health's official website was unable to provide information regarding the accumulated data on June 6, 2020 (Carmo; Caneppele; Nunes, 2021; Idrovo; Manrique-Hernández; Nino, 2021; Silva; Figueiredo Filho, 2021).

In this context, press vehicles, state and municipal health secretariats, and specialized websites were established to disseminate information regarding the COVID-19 pandemic in Brazil (Carmo; Caneppele; Nunes, 2021; Idrovo; Manrique-Hernández; Nino, 2021). As a result, several studies have been conducted, supporting decision-making on how to deal with the SARS-CoV-2 coronavirus crisis.

Since its first use for assessing the quality of data from public surveillance systems regarding the influenza A (H1N1) pandemic in 2009 (Idrovo; Manrique-Hernández; Nino, 2021), the NB Law has been recurrently used to identify indications that public health surveillance systems are functioning properly (Kennedy; Yam, 2020), which has gained even more attention with the emergence of the COVID-19 pandemic. For, by taking into account the costs involved in the processes of investigations into the quality of these systems, analysis from methods capable of flagging anomaly numbers allows health authorities to direct their attention to those activities generating suspicious data (Kennedy; Yam, 2020).

In this sense, specifically, when applied to the first digits of the numbers generated by epidemiological surveillance systems, the NB Law could be considered a fast and inexpensive tool and could be used for the assessment of the informational quality of these systems (Idrovo; Manrique-Hernández, 2020).

Despite the opinions of researchers who advocate the application of the NB Law for assessing the quality of data generated by epidemiological surveillance systems, Druică, Oancea, and Vâlsan (2018) draw attention to some limitations of this analytical methodology aimed at identifying anomaly numbers, namely: there are certain data sets that, due to their nature, do not conform to the NB Law; the statistical tests used to detect violations of the NB Law are not always 100% reliable; because they suffer influence from the size of the respective samples; the null hypothesis significance tests (NHST) and the mean absolute deviation (MAD), as evaluative metrics of the adequacy and conformity to the NB Law, may present conflicting results.

Regardless of limitations, studies on the appropriateness of adjusting pandemic COVID-19 data to the NB Law have shown divergent results, even when conducted with considerable numbers of observations and with a significant diversity of periods and health information systems around the world. Thus, the number of surveys that identified non-compliance (18 occurrences) with the NB Law is far greater than those researches in which compliance was found (8 occurrences), as demonstrated by the data summarized in Table 2.

Table 2. Analysis of the COVID-19 pandemic data for compliance with the NB Law

Reference	Adaptation to NB Law	Z Test	KS Test	X^2 Test	MAD
Kennedy and Yam (2020)	conformity	no	no	yes	no
Idrovo and Manrique-Hernández (2020)	conformity	no	no	yes	no
Hurtado-Ortiz *et al*. (2020)	conformity	no	no	yes	no
Koch and Okamura (2020)	conformity	no	no	yes	no
Miranda (2020a)	conformity	no	yes	no	no
Miranda (2020b)	conformity	no	yes	no	no
Pahuja (2021)	conformity	no	no	yes	no
Guliyev (2021)	conformity	no	yes	yes	no
Carmo, Caneppele, and Nunes (2021)	non-conformity	yes	no	no	yes
Galveas, Barros Jr, and Fuzo (2021)	non-conformity	yes	no	yes	yes
Idrovo, Manrique-Hernández, and Nino (2021)	non-conformity	no	no	yes	no
Silva and Figueiredo Filho (2021)	non-conformity	no	no	yes	yes
Adam and Tsarsitalidou (2022)	non-conformity	no	no	no	yes
Campolieti (2021)	non-conformity	no	no	no	yes
Natashekara (2021)	non-conformity	yes	yes	yes	yes
Kilani and Gergiou (2021)	non-conformity	no	no	yes	yes
Morillas-Jurado, Caballer-Tarazona, and Caballer-Tarazona (2022)	non-conformity	no	no	yes	no
Kolias (2022)	non-conformity	no	yes	yes	yes
Isea (2020)	non-conformity	no	no	yes	no
Žmuk e Jošić (2020)	non-conformity	no	yes	yes	no
Moreau (2021)	non-conformity	no	no	yes	no
Farhadi (2021)	non-conformity	no	yes	yes	no
Mahasuar (2021)	non-conformity	no	yes	yes	yes
Fatima, Khaliq, and Younas (2021)	non-conformity	no	no	yes	no
Farhadi and Lahooti (2021)	non-conformity	no	yes	yes	yes
Kilani (2021)	non-conformity	no	no	yes	yes

Source: elaborated by the authors, based on the research data.

According to Table 2, it is possible to observe some variety concerning the types of statistical tests used to evaluate the adequacy of the data concerning the pandemic of COVID-19 about the NB Law. Among other less used tests, the most recurrent are the Z test (3 occurrences), the KS test (9 occurrences), the X^2 test (21 occurrences), and the MAD (11 occurrences).

In this sense, part of the existing diversity concerning the results of studies involving the application of the Law of NB to pandemic COVID-19 data could be attributed not only to the quality of the data itself, but also to the statistical tests used to detect possible violations, as suggested by Druică, Oancea, and Vâlsan (2018). The explanation for such a hypothesis is because, as metrics used to assess adjustments to the NB Law, the null hypothesis significance tests (NHST = Z, KS, and X^2 tests) and the mean absolute deviation (MAD) may present results that conflict with each other, since the former tend to be influenced by the size of the sample analyzed.

The Z test aims to evaluate the hypothesis of significant difference between proportions from comparisons between the observed relative frequency (*Orf*) and the expected relative frequency (*Erf*), becoming more rigorous for numerous samples (n) (Costa et al., 2012), as can be seen in Equation 2.

$$Z = \frac{Orf - Erf}{\sqrt{\frac{Orf\,(1 - Orf)}{n}}} \tag{2}$$

The KS test is a non-parametric test based on the absolute maximum difference between two cumulative distributions of functions, that is, an empirical function ($F_{1,n}$) and a hypothetical function ($F_{2,m}$) (Wilks, 2006). Still, a cumulative observed relative frequency (*Orf*) and an expected cumulative relative frequency (*Erf)*, whose statistics can be calculated from Equation 3.

$$D_{n,m} = supX\left|F_{1,n}(x) - F_{2,m}(x)\right| \tag{3}$$

In Equation 3, $F_{1,n}$, and $F_{2,m}$ are cumulative distribution functions (*Erf* and *Orf*), and *supX* is the supremum function given by the largest deviation between the compared cumulative frequencies. Thus, the equality hypothesis is rejected for a significance level α if the behavior described by Equation 4 is observed, in which, n and m are the sizes of the first and second samples (*Erf* and *Orf*, respectively). Where the value of α for the most common significance levels, thus 0.10, 0.05, 0.025, 0.01, 0.005, and 0.001, are 1.22, 1.36, 1.48, 1.63, 1.73, and 1.95, respectively (Montgomery; Runger, 2010; Soong, 2004).

$$D_{n,m} > c(\alpha)\sqrt{\frac{n + m}{n\,m}} \tag{4}$$

Proposed by Pearson (1898) and used for comparisons between the behavior of an observed frequency for a given sample and an expected frequency, the X^2 test can also be used to assess whether the observed behavior in a series of data fits the expected behavior according to a given theoretical probability or law of occurrence (Montgomery; Runger, 2010). Thus, as described in Equation 5, from the comparison between the observed frequency distribution (*Of*) and the expected frequency (*Ef* for each category/class of values (i), therefore Of_i and Ef_i, respectively, and also taking into account the respective quantities of expected occurrences (ei), the X^2 test indicates whether there is a significant difference between observed and expected values (Siegel; Castellan, 2008).

$$X^2 = \sum_{i=1}^{k} \frac{(Of_i - Ef_i)^2}{e_i} \tag{5}$$

From the calculation described by Equation 5, the calculated value (critical X^2) is compared with the tabulated value (tabulated X^2) according to the desired significance level (α) and the respective number of degrees of freedom (*df*), given by *n* minus one observation, i.e.: *df* = n - 1 (Montgomery; Runger, 2010).

The MAD is described as the average of the absolute deviations occurring for a set of category/class (*k*) of observations. And, in the specific case of its application to the evaluation of the conformity of the NB Law, it can be calculated from the division of the sum of the deviations (differences) between the observed (*Orf*) and expected (*Erf*) frequencies, for each digit, by the total (*k*) digits analyzed (Mcclave; Benson; Sincich, 2005), as described by Equation 6.

$$MAD = \frac{\sum_{i=1}^{k} |Orf_i - Erf_i|}{k} \tag{6}$$

In contrast to the null hypothesis significance tests (NHST), which are those whose null hypothesis implies the equality of frequencies of the compared series, as is the case of the Z, KS, and X^2 tests that take into account the number of observations integrating the respective samples, the MAD presents as a limitation the fact that it does not have significance intervals for analysis, as happens with NHST (Nigrini, 2012). To overcome this deficiency, Nigrini (2012) proposes a scale of values to assess compliance levels concerning the NB Law, as shown in Table 3.

Table 3. Critical values for analyzing the mean absolute deviation from NB Law

Digit	Full conformity	Acceptable conformity	Marginal conformity	Non-conformity
First	0.000 to 0.006	0.006 to 0.012	0.012 to 0.015	Higher than 0.015
Second	0.000 to 0.008	0.008 to 0.010	0.010 to 0.012	Higher than 0.012
First two digits	0.000 to 0.012	0.012 to 0.018	0.018 to 0.022	Higher than 0.022

Source: elaborated by the authors, based on the research data.

Therefore, considering that Druică, Oancea and Vâlsan (2018) suggested that tests used in evaluating the adjustment to the NB Law may affect the results of investigations involving financial data, and also, considering that this study uses samples pertaining to Brazilian federation units (27 observations) and municipalities (5,570 observations), after two years of arrival of the COVID-19 pandemic. As a result, in a cross-sectional and temporal perspective different from that of Carmo, Caneppele and Nunes (2021), this study is expected to contribute new scientific findings regarding both the limitations of the NB Law and the statistical tests used in evaluating data and information published by the Brazilian health system about the cases of contaminations and deaths caused by the SARS-CoV-2 coronavirus.

2. Materials and Methods

Considering that the data made available by the Brazilian government (Brasil, 2022a) may have been compromised by the change of criteria and interruption in the flow of information that occurred on June 6, 2020 (Carmo; Caneppele; Nunes, 2021; Idrovo; Manrique-Hernández; Nino, 2021; Silva; Figueiredo Filho, 2021), herein, the sample was extracted from the information base made available by the public data repository Brasil.IO (2022). Since that interruption, Brasil.IO has been compiling, storing, and reporting daily the data of confirmed cases and deaths by COVID-19 obtained from the bulletins of the State Health Secretariats (SES) in Brazil, among other information.

In this sense, the first sample of this research was formed with the data referring to the total number of confirmed cases and the total number of deaths that occurred until 02/25/2022, both due to contamination by the SARS-CoV-2 coronavirus. Therefore, with data accumulated over two years since the arrival of the COVID-19 pandemic in Brazil. The second sample is regarding the total number of confirmed cases and the total number of deaths reported exactly on 02/25/2022. Thus, it is about the specific data of the day when two years had passed since the arrival of the SARS-CoV-19 pandemic in Brazil.

The justification for using two different samples resides in the possibility that the first series of data (accumulated) carries with it the reflections of informational failures that occurred at different times during the COVID-19 pandemic in Brazil over the two years analyzed. On the other hand, the analysis referring to data from a single day, as is the case of the second sample (02/25/2022), would allow us to evaluate if the Brazilian information system would be working adequately, in the light of the NB Law, after two years of the pandemic, and also without the influence of failures which occurred during this time interval.

After tabulating the data extracted from the Brasil.IO repository (2022) at the federal and municipal levels, the first sample (with accumulated data) was composed of 27 observations referring to the total contamination and 27 observations concerning the total number of deaths, both for the 26 states and the federal district, and also 5,570 observations referring to the total contamination and 5,545 observations referring to the total number of deaths, both related to Brazilian municipalities. The second sample (with specific data for 02/25/2022) was composed of 27 observations regarding the total contamination and 25 observations regarding the total number of deaths for the 26 states and the federal district, and also 538 observations regarding the total contamination and 65 observations regarding the total number of deaths, both referring to Brazilian cities whose information was available for 02/25/2022.

Initially, the observed absolute (*Of*) and relative (*Orf*) frequencies of occurrence were calculated, referring to the first digits of the totals informed for the data integrating those two samples. Next, the respective expected frequencies, absolute (*Ef*) and relative (*Erf*) were calculated, according to the NB Law. Finally, the adjustment of the observed frequencies to the expected frequencies according to the NB Law was evaluated by applying the Z, KS, and X^2 (NHST) tests. Furthermore, the respective MADs were calculated, and their level of adjustment to the Law of NB was evaluated using the compliance scale proposed by Nigrini (2012).

The entire process of identifying and analyzing the first digits, calculating and tabulating frequencies, as well as implementing the Z, KS, and X^2 tests, and calculating the MAD was performed using spreadsheets and a computer with an Intel® Core™ i3-1005 G1 processor, CPU @ 1.20 GHz and 1.19 GHz, with 4.00 GB of RAM installed and Windows 10 64-bit operating system.

Thus, concerning its object of study, the respective analytical methods, and the nature of the data analyzed, this study can be classified as scientific research of an empirical nature based on applied quantitative methods.

3. Results and Discussion

When analyzing the series related to the data of contaminations and deaths accumulated over the 2 years of the pandemic of COVID-19 in Brazil, totaled for its 27 federative units, those 3 NHSTs used in this research signaled the occurrence of adjustment (conformity) to the expected frequencies according to the NB Law. As can be observed by the data summarized in Table 4, with emphasis on the Z test (p-value > 0.05), which is the only one that allows analyzing the conformity according to each of the first 9 digits analyzed.

Table 4. Data accumulated until 02/25/2022 referring to the federation units

1st Digit	Contaminations						Deaths					
	Of	Ef	Orf	Erf	Est. Z	p-value	Of	Ef	Orf	Erf	Est. Z	p-value
1	9	8	0.3333	0.3010	0.366	0.36	10	8	0.3704	0.3010	0.785	0.22
2	4	5	0.1481	0.1761	-0.381	0.65	7	5	0.2593	0.1761	1.135	0.13
3	4	3	0.1481	0.1249	0.365	0.36	1	3	0.0370	0.1249	-1.381	0.92
4	4	3	0.1481	0.0969	0.900	0.18	2	3	0.0741	0.0969	-0.401	0.66
5	2	2	0.0741	0.0792	-0.098	0.54	1	2	0.0370	0.0792	-0.811	0.79
6	1	2	0.0370	0.0669	-0.622	0.73	2	2	0.0741	0.0669	0.148	0.44
7	2	2	0.0741	0.0580	0.358	0.36	3	2	0.1111	0.0580	1.181	0.12
8	1	1	0.0370	0.0512	-0.333	0.63	1	1	0.0370	0.0512	-0.333	0.63
9	0	1	0.0000	0.0458	-1.138	0.87	0	1	0.0000	0.0458	-1.138	0.87
Total	27	27	1.0000	1.0000			27	27	1.0000	1.0000		

Source: elaborated by the authors, based on the research data.

And in this case, we accept H_0 and assume an adjustment between observed and expected frequency. For COVID-19 deaths, we observed a KS test statistic of 0.1525 against a critical KS value of 0.2617 (for n=27 and 0.05 significance level), so KS test < critical KS, which also allows us to accept H_0 and assume the adjustment between observed and expected frequency.

Regarding the X^2 test, both for contaminations and deaths, the respective statistics corroborated with the results of the Z and KS tests, and thus it was possible to assume the adjustment between the observed frequency and the expected frequency since for the cases of contaminations the X^2 test statistic was 2.89 (p-value p > 0.05), and for the deaths, the X^2 test statistic was 6.59 (p-value p > 0.05).

Contrary to the results identified by the NHST, the MAD indicated non-compliance with respect to the NB Law, since, for contamination cases a MAD of 0.027 (MAD > 0.015) was observed, and for COVID-19 deaths a MAD of 0.047 (MAD > 0.015) was observed.

Regarding the data accumulated until 02/25/2022 related to the information at the municipal level, the occurrence of conformity according to the NB Law was only observed for the KS test applied to deaths. For, as presented in Table 5, the Z Test indicated differences (p-value < 0.05) for the totals with first digits 5, 6, and 9, in the case of contaminations, and also, 7 and 8 in the case of deaths.

Table 5. Data accumulated until 02/25/2022 referring to municipalities

1st Digit	Contaminations						Deaths					
	Of	Ef	Orf	Erf	Est. Z	p-value	Of	Ef	Orf	Erf	Est. Z	p-value
1	1609	1677	0.2889	0.3010	-1.979	0.98	1708	1669	0.3080	0.3010	0.911	0.18
2	975	981	0.1750	0.1761	-0.205	0.58	936	976	0.1688	0.1761	-1.573	0.94
3	683	696	0.1226	0.1249	-0.523	0.70	691	693	0.1246	0.1249	-0.198	0.58
4	524	540	0.0941	0.0969	-0.715	0.76	490	537	0.0884	0.0969	-2.250	0.99
5	515	441	0.0925	0.0792	3.670	0.00	417	439	0.0001	0.0792	-1.190	0.88
6	411	373	0.0738	0.0669	2.043	0.02	364	371	0.0656	0.0669	-0.476	0.68
7	311	323	0.0558	0.0580	-0.689	0.75	357	322	0.0644	0.0580	1.944	0.03
8	257	285	0.0461	0.0512	-1.698	0.96	316	284	0.0570	0.0512	1.886	0.03
9	285	255	0.0512	0.0458	1.932	0.03	266	254	0.0480	0.0458	0.712	0.24
Total	5570	5570	1.0000	1.0000			5545	5545	1.0000	1.0000		

Source: elaborated by the authors, based on the research data.

Furthermore, with the accumulated data until 02/25/2022 regarding municipal information (Table 5), the KS test showed a KS statistic of 0.0184 against a critical KS of 0.0182 (for n=5570 and 0.05 significance level). Therefore, test KS > critical KS, which led to the rejection of H_0 and the assumption of a difference between the frequency of observed data compared to the respective expected frequency concerning cases of contamination by COVID-19. Regarding the cumulative deaths until 02/25/2022 related to municipal information, the KS test showed a KS statistic of 0.0144 against a critical KS value of 0.0183 (for n=5545 and 0.05 of significance), therefore, test KS < critical KS, indicating adjustment between the observed data series and the expected series according to the NB Law, as previously stated.

Regarding the X^2 test, both for contaminations and deaths, the respective statistics also indicated non-compliance with the NB Law, since for the cases of contaminations the X^2 test statistic was 26.51 (p-value < 0.05), and for deaths, the X^2 test statistic was 16.19 (p-value < 0.05).

Finally, the MAD indicated full compliance (MAD < 0.006) concerning the NB Law for those two data series described in Table 5. That is, a MAD of 0.0057 was observed for COVID-19 contaminations and a MAD of 0.0048 for deaths, both reported cumulatively by the municipalities until 2/25/2022.

When analyzing the data referring to 02/25/2022, therefore, exclusively on the day when the pandemic COVID-19 completed two years of its arrival in Brazil, it could be observed that the Z test signaled nonconformity regarding the totals informed by the Federal District and Brazilian states with the first digit equal to 9 (p-value < 0.05). Concurrently, presented conformity about the other first digits (p-value > 0.05) of the totals referring to cases of contamination by COVID-19, as well as, concerning all the first digits of the totals referring to deaths reported by those states, as shown in Table 6.

Table 6. Exclusive data of 02/25/2022 referring to the federation units

1st	Contaminations						Deaths					
Digit	Of	Ef	Orf	Erf	Est. Z	p-value	Of	Ef	Orf	Erf	Est. Z	p-value
1	8	8	0.2963	0.3010	-0.054	0.52	10	8	0.4000	0.3010	1.079	0.14
2	7	5	0.2593	0.1761	1.135	0.13	5	4	0.2000	0.1761	0.314	0.38
3	1	3	0.0370	0.1249	-1.381	0.92	2	3	0.0800	0.1249	-0.680	0.75
4	2	3	0.0741	0.0969	-0.401	0.66	1	2	0.0400	0.0969	-0.962	0.83
5	2	2	0.0741	0.0792	-0.098	0.54	2	2	0.0800	0.0792	0.015	0.49
6	2	2	0.0741	0.0669	0.148	0.44	2	2	0.0800	0.0669	0.261	0.40
7	1	2	0.0370	0.0580	-0.466	0.68	0	1	0.0000	0.0580	-1.241	0.89
8	0	1	0.0000	0.0512	-1.206	0.89	2	1	0.0800	0.0512	0.655	0.26
9	4	1	0.1481	0.0458	2.546	0.01	1	1	0.0400	0.0458	-0.138	0.55
Total	27	27	1.0000	1.0000			25	25	1.0000	1.0000		

Source: elaborated by the authors, based on the research data.

Still, regarding the data of the totals informed by the federative units of Brazil referring exclusively to 02/25/2022, the KS test signaled conformity concerning the NB Law. Since for the cases of contamination by COVID-19, there is a KS statistic of 0.1024 against a critical KS value of 0.2617 (for n=27 and 0.05 significance level), therefore, test KS < critical KS, which allows accepting H_0 and assuming adjustment. And further, for COVID-19 deaths, a KS test statistic was observed with a value of 0.1229 against a critical KS value of 0.2720 (for n=25 and 0.05 significance), so one has a test KS < critical KS, which allows one to accept H_0 and assume adjustment between expected frequency and observed frequency.

Similarly, to the KS test, the X^2 test also showed compliance with the NB Law for both contamination and deaths related to the totals informed by the federative units of Brazil exclusively for February 25, 2022: for contamination cases, the X^2 test statistic was 10.68 (p-value > 0.05); and for deaths, the X^2 test statistic was 4.07 (p-value > 0.05).

On the other hand, the calculation of the MAD with the totals informed by the federative units of Brazil referring exclusively to February 25, 2022, had an inverse behavior to the NHST. The MAD for the cases of contamination was 0.043, indicating a lack of conformity with the NB Law (MAD > 0.015), and the MAD for the deaths was 0.037, also indicating non-conformity with the NB Law (MAD > 0.015).

As can be seen from the information summarized in Table 7, the analysis of the data reported by the Brazilian municipalities showed non-compliance with the NB Law for the first digits 1 and 2 (Z-test p-value < 0.05), in the case of COVID-19 contamination, and also for the first digit 1 (Z-test p-value < 0.05), in the case of deaths.

Table 7. Exclusive data from 02/25/2022 referring to municipalities

1st	Contaminations						Deaths					
Digit	*Of*	*Ef*	*Orf*	Erf	Est. Z	p-value	Of	Ef	Orf	Erf	Est. Z	p-value
1	184	162	0.3420	0.3010	2.072	0.02	50	20	0.7692	0.3010	8.229	0.00
2	109	95	0.2026	0.1761	1.614	0.05	9	11	0.1385	0.1761	-0.796	0.79
3	69	67	0.1283	0.1249	0.232	0.41	4	8	0.0615	0.1249	-1.546	0.94
4	48	52	0.0892	0.0969	-0.603	0.73	0	6	0.0000	0.0969	-2.641	1.00
5	38	43	0.0706	0.0792	-0.734	0.77	0	5	0.0000	0.0792	-2.364	0.99
6	33	36	0.0613	0.0669	-0.520	0.70	1	4	0.0154	0.0669	-1.663	0.95
7	20	31	0.0372	0.0580	-2.066	0.98	1	4	0.0154	0.0580	-1.470	0.93
8	17	28	0.0316	0.0512	-2.059	0.98	0	3	0.0000	0.0512	-1.872	0.97
9	20	25	0.0372	0.0458	-0.953	0.83	0	3	0.0000	0.0458	-1.765	0.96
Total	538	538	1.0000	1.0000			65	65	1.0000	1.0000		

Source: elaborated by the authors, based on the research data.

The KS test showed divergent results from the Z test applied to the total data for the Brazilian municipalities. That is, for cases of contamination by COVID-19 we observed a KS test statistic of 0.0708 against a critical KS of 0.0586 (for n=538 and 0.05 significance level), i.e., test KS > critical KS, which leads to the rejection of H_0, therefore, a difference between the observed and the expected data series is assumed. Regarding COVID-19 deaths, a KS test statistic of 0.4682 was observed against a critical KS value of 0.0169 (for n=65 and 0.05 significance), thus test KS > critical KS, rejecting H_0 and therefore assuming the existence of a difference between the observed and the expected data series.

The X^2 test indicated compliance according to the NB Law for cases of COVID-19 contamination (X^2 test statistic was 15.18, p-value > 0.05) and divergence between the observed and expected data series for deaths (X^2 test statistic was 72.31, p-value < 0.05).

Unlike NHST, whose results were divergent from each other, both the MAD calculated cases of contamination by COVID-19 and deaths, both at the municipal level, presented non-compliance concerning the NB Law. That is, the MAD for contaminations was 0.0157, which indicates no compliance with the NB Law (MAD > 0.015), and the MAD for deaths was 0.1040, which indicates non-compliance with the NB Law (MAD > 0.015).

As it can be seen, there are indications that the amount of data in the samples analyzed influences the results of the tests used to evaluate the existence of possible non-conformities resulting from the application of the NB Law in the evaluation of the informational quality of Brazilian epidemiological monitoring systems regarding the cases of contamination and death caused by the SARS-CoV-2 coronavirus.

In this sense, concerning the data accumulated over the two years since the arrival of the COVID-19 pandemic in Brazil, there is a certain convergence of results with NHST. Furthermore, contraposition of the results of these tests about MAD, as can be seen in the summary described in Table 8.

Table 8. Analysis of compliance with the NB Law according to samples and tests

Test	Sample 1: accumulated data				Sample 2: data from 02/25/2022			
	Contaminations		Deaths		Contaminations		Deaths	
	FU (n= 27)	Mun. (n= 5570)	FU (n= 27)	Mun. (n= 5545)	FU (n= 27)	Mun. (n= 538)	FU (n= 25)	Mun. (n= 65)
Z Test	Yes	No	Yes	No	No	No	Yes	No
KS Test	Yes	No	Yes	Yes	Yes	No	Yes	No
X^2 Test	Yes	No	Yes	No	Yes	Yes	Yes	No
MAD	No	Yes	No	Yes	No	No	No	No

Source: elaborated by the authors, based on the research data.

Otherwise, for the data unique to 2/25/2022, that convergence is not as well defined concerning NHST, while the MAD appeared to be more uniform to that particular sample, as can be seen from the information contained in Table 8.

Thus, it seems reasonable to consider the warning made by Druică, Oancea, and Vâlsan (2018) about the need to consider the effect of the variables time and sample size on the results of compliance tests concerning the NB Law. For, both in studies in a longitudinal direction, as in the case of Carmo, Caneppele, and Nunes (2021), and in a cross-sectional direction, as in the case of most of the studies already conducted. Also, in the case of the present investigation, the four tests used seem to present conflicting results among themselves, even when applied to the same sample.

Additionally, we highlight the fact that studies focusing on the use of the NB Law as a tool for assessing the quality of the functioning of epidemiological health information systems during the COVID-19 pandemic have also shown conflicting results around the world, as described earlier in Table 2 of the present study.

If on the one hand the divergences identified among studies with different samples (see Table 2 of the theoretical framework) could be attributed to factors of the most varied natures (political, economic, social, etc.). Still, to the quality of the public health information systems themselves. On the other hand, it is not possible to state that such divergences in results could not also be attributed to the tests used to evaluate the levels of adequacy and compliance with the NB Law. Since here, divergences in results were evidenced between those four distinct tests, even when applied to the same samples.

Regardless of possible divergence of results due to the operationalization of the statistical tests used to assess the compliance of Brazilian public health information systems during the pandemic of COVID-19, based on the NB Law. The fact is that at least one of them signaled the existence of possible non-conformities both between the federative units and the country's municipalities.

4. Conclusions

Analyzing possible divergences in the results from the application of the NB Law for the evaluation of the conformity of the information concerning the cases of contamination and deaths caused by the SARS-CoV-2 coronavirus, from a transversal cut performed two years after the arrival of the COVID-19 pandemic in Brazil, this research compared 3 null hypothesis significance tests (NHST), that is, the Z test, the X^2 test, and the KS test, and also the mean absolute deviation (MAD).

Regarding the conformity of the Brazilian information on cases of contamination and deaths caused by the SARS-CoV-2 coronavirus, at least one of the tests used signaled non-conformity, when this was not signaled by two or more tests simultaneously. This finding can be considered a strong indication that the Brazilian epidemiological information system did not work adequately during the COVID-19 pandemic, and that there was no improvement in quality even after two years.

Compared to the results of previous studies on the subject, conducted nationally and globally, the variability of the results of statistical tests observed in this paper, even when applied to the same sample, provides evidence that, in addition to the need to take into account the effect of time and sample size variables on the results of compliance tests concerning the NB Law (Druică; Oancea; Vâlsan, 2018). Furthermore, it can be admitted that the data series related to information about contaminations and deaths caused by the SARS-CoV-2 coronavirus present distinct behavior from the probability distribution of frequencies according to the NB Law. Whether due to a natural process or logistical issues, data distortions, forms of geographic aggregation, errors, and/or negligence, among other factors, as already suggested by Carmo, Caneppele, and Nunes (2021).

As a suggestion for the continuity of this study, it is recommended that it be replicated, however, expanding the type and quantity of statistical tests used to evaluate the conformity of the data series arising from the COVID-19 pandemic to the NB Law.

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Conflicts of Interest

The authors declare no conflict of interest.

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