Brazilian Journal of Biometrics, **41**, 424–444 (2023)
doi:10.28951/bjb.v41i4.665

BRAZILIAN JOURNAL OF
BIOMΣTRICS
ISSN:2764-5290

ARTICLE

Corn plant dry mass accumulation considering the previous crop by non-linear models

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(Received: March 22, 2023; Revised: April 10, 2023; Accepted: May 10, 2023; Published: December 1,2023) [1]

Abstract

Corn is the most produced cereal in the world, used both in human and animal nutrition. This study aimed to compare the fit of the Logistic, Gompertz, and von Bertalanffy non-linear models to data on the accumulation of total dry mass, dry mass of stems, leaves, and ears in corn plants grown with straw mulch from common bean, millet, and Brachiaria brizantha in relation to the days after plant emergence. The assumptions of normality, homoscedasticity, and independence of residuals were checked by Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson tests, respectively. The models were adjusted by the least squares method using the Gauss-Newton algorithm in the R software. The quality of the fit was evaluated based on the values of the coefficient of determination (R2), the residual standard deviation (RSD), the Akaike Information Criterion (AIC), and Bates and Watts curvature measures. The Logistic model presented the best fit for the dry mass of stems and ears, and the Gompertz model for the dry mass of leaves and total dry mass, based on the quality evaluators used.

Keywords: Brachiaria brizantha; Common bean; Millet; Regression.

1. Introduction

Corn is one of the main crops in Brazil, cultivated practically throughout the national territory, and used both in human and animal nutrition, due to its high nutritional value. The leading corn producers in the world in order are the USA, China, and Brazil (USDA, 2023). Total production in the 2021/22 growing season in Brazil was 116 million tons of grains, in which the largest producing states were MT, PR, MS, and GO (IBGE; CONAB, 2022). Thus, this crop is very important to the Brazilian agribusiness, contributing both to the economic and social sectors of the country. However, despite its great importance for the Brazilian agribusiness, corn productivity is still considered low. A major factor accounting for the low productivity of corn is the improper management of nitrogen, whose efficiency of use by the plant is influenced by the cultivation system, type of fertilizer, and management methods used (Rasteiro *et al.*, 2020).

Corn production in Brazil is characterized by planting in two seasons: first crop (or summer crop) and second crop (or off-season). The summer crop corn is planted between October and December, while the off-season corn can be grown between January and April. The second crop is called off-season because it is not planted in an ideal period for the crop, and water deficits may occur, also because there is less sunlight in some regions of the country (Cunha *et al.*, 2019). Off-season planting has a great contribution to the growth of corn production in Brazil, and in 2022 the production in the country recorded approximately 25 million tons in the summer harvest and 85 million tons in the off-season harvest (IBGE; CONAB, 2022).

For corn, it is fundamental to define the system to be used in the planting, whether the Conventional Planting System or the No-Till Planting System. The first uses traditional practices for soil preparation, all vegetation on the land is removed, and the soil is stirred by plowing or harrowing (Fagundes *et al.*, 2019). In turn, the No-Till Planting System is the sowing technique in which the seed is placed in undisturbed soil (without prior plowing or harrowing). Considered an innovative practice, a management alternative that advocates minimal soil disturbance, the presence of straw mulch on the soil surface, and crop rotation (Salomão *et al.*, 2020).

Several crops can be used as soil straw mulch in corn planting, including the common bean, which is a short-cycle plant and is usually grown at various times of the year (Lima *et al.*, 2017). Pearl millet is a good alternative grass for forage production, as it is easy to plant and manage, with easy adaptation to different climates and soil conditions (Pinho *et al.*, 2013). Another plant widely used as straw mulch is *Brachiaria brizantha*, a forage grass with high economic importance in Brazil; due to its relatively easy management, it has become responsible for the advancement of Brazilian livestock, and also offering pastures of reasonable quality and moderate investments (Silva *et al.*, 2022).

Several types of statistical models can be applied to study the growth processes in plant production systems. Studies have shown that plant growth and biomass accumulation have a sigmoidal behavior described by non-linear models. These models offer useful information and estimates, as they allow practical interpretation of the parameters, especially about biological processes (Jane *et al.*, 2020).

The parameters of plant growth models allow inferring the growth and development rate, the degree of maturity, and the maximum cumulative content of nutrients. The information obtained can contribute to the proper management of the crop, thus enabling the detection of factors that impair development, such as suitable time for topdressing fertilization, occurrences of water and nutritional deficiencies, the incidence of pests and diseases, as well as unfavorable weather conditions. The factors that have most affected corn productivity are climate, nutrient management, soil fertility, crop management practices, the genetic potential of materials, and pest and disease management (Oliveira *et al.*, 2013).

Understanding the growth and development of the corn plant and the behavior of variables involved in the process are important in the search for increased crop productivity. Thus, the objective of this study was to characterize the agronomic performance of corn, describing the accumulation of total dry mass, dry mass of stems, leaves, and ears of corn in relation to the days after emergence of plants grown on straw mulch from common bean, *Brachiaria brizantha*, and millet, comparing the Logistic, Gompertz and von Bertalanffy non-linear models and indicating the most appropriate model.

2. Material and Methods

The data used to fit the models were extracted from Oliveira *et al.* (2013). The experiment was carried out in the summer of the 2007/2008 agricultural year, at Capivara Farm, located in the municipality of Santo Antônio de Goiás, state of Goiás. In the winter of 2007, common bean, *B. brizantha*, and pearl millet were cultivated to be used as cover crops for corn, in the summer of 2007/2008. This was a randomized complete block experimental design, with three treatments (common bean, *B. brizantha*, and pearl millet straw mulches) and five replications, where the average of the treatments was used for analysis.

For growth analysis, plants were sampled every seven days, starting 20 days after emergence (DAE), with the collection of two plants per plot, which were placed in plastic bags and taken to the laboratory, measuring the accumulation of total dry mass (gram/m^2), and accumulation of dry mass of leaves, stems and ears.

The first step was to fit non-linear models to the data and check whether the assumptions were met in the residual analysis using the Shapiro-Wilk, Breush-Pagan, and Durbin-Watson tests. In case of heteroscedasticity of variances, new fits were made by weighed estimates, incorporating heteroscedasticity at the moment of estimating the model parameters.

To analyze and model corn plant growth and productivity, the non-linear regression models used were: Logistic $Y_i = \frac{\alpha}{1+e^{\kappa(\beta-x_i)}} + \varepsilon_i$, Gompertz $Y_i = \alpha e^{-e^{\kappa(\beta-x_i)}} + \varepsilon_i$ and von Bertalanffy $Y_i = \alpha\left(1 - \frac{e^{\kappa(\beta-x_i)}}{3}\right)^3 + \varepsilon_i$,

where, $i = 1, 2, ..., n$; Y_i is the i-th observation of the dependent variable; x_i is the i-th observation of the independent variable; α is the expected value for the maximum growth under study; β is the abscissa of the inflection point, at which growth slown down, κ is an index associated with growth, the higher its value, the less time is needed for the object under study to reach its inflection point; εi is the random errors attributed to the model associated with the i-th observation, assuming that it is independently and identically distributed following a normal distribution of zero mean and constant variance, that is, $\varepsilon_i \sim N(0;\sigma^2)$. The inflection point (PI) of the Logistic model is given by $(\beta, \alpha/2)$, of the Gompertz model is given by $(\beta, \alpha/e)$ and of the von Bertalanffy model $(\beta,(2/3)^3\alpha)$. The point of maximum acceleration (PAM) of the Logistic model is $((k\beta - 1.319)/k, \alpha/(3+\sqrt{3}))$, of the Gompertz model is

$((k\beta - 0.962)/k,\ \alpha e^{(-2+\sqrt{5})/2})$ and the point of maximum deceleration (PDM) of the Logistic model is $((k\beta + 1{,}319)/k,\ \alpha/(3 - \sqrt{3}))$, and of the Gompertz model is $((k\beta + 0.962)/k,\ \alpha e^{(-2-\sqrt{5})/2})$.

The estimation of the model parameters was based on the least squares method, using the Gauss-Newton convergence algorithm, in which the initial values for the parameters were obtained through a graphical analysis of the behavior of the function.

Residuals were analyzed for the assumptions of normality, homogeneity, and independence of the model, at a 5% level of significance, using the Shapiro-Wilk, Breuch-Pagan, and Durbin-Watson tests, respectively.

With the fits made, the analyses were carried out and when the assumption of normality was met, the confidence intervals were constructed, in violation of the assumption of independence of residuals, new fits were made to the models with the inclusion of the first order autoregressive term (AR1) to obtain more accurate estimates of the parameters, and to improve the quality of the fit.

In case of violation of homoscedasticity assumption, the weighting factor was estimated using the “weights” argument of the “gnls” function of the R software (R Core Team, 2023), the tested classes were "VarExp()", "VarIdent()", "VarPower()" and the class with the lowest AIC value was selected.

Student’s t-test was applied, with $v = n\text{-}p$ degrees of freedom, for the significance of parameters α, β, and κ, in which the null hypothesis (H_0) refers to the parameter being equal to zero, and the alternative hypothesis (H_a) refers to the parameter being different from zero. The 95% probability confidence intervals were also obtained.

The evaluation and comparison of the goodness of fit of the model that best describes the data were performed using the coefficient of determination (R^2), the residual standard deviation (RSD), Akaike information criterion (AIC), and Bates and Watts curvatures, which measure the non-linearity of the model. The best model provided the highest R^2 value and the lowest values for the RSD, AIC. By using the Bates and Watts curvatures, the best model was the one with the lowest values of intrinsic (c^J) and parametric (c^θ) non-linearity.

All analyses were performed using the free R statistical software (R Core Team, 2023). The packages used were nlme (version: 3.1-162, Pinheiro; Bates, 2023), car (version: 3.1-1, Fox; Weisberg, 2019), lmtest (version: 0.9-40, Zeileis; Hothorn, 2002), and qpcR (version: 1.4-1, Spiess, 2018).

3. Results and Discussion

3.1 Dry mass of stems

After fitting the Logistic, Gompertz and von Bertalanffy models to the data, considering that all assumptions about the error vector are adopted, that is, the residuals are independent and identically distributed following a zero mean normal and constant variance ($\varepsilon \sim N(0, \mathbf{I}\sigma^2)$), residuals were analyzed by Shapiro-Wilk, Durbin-Watson, and Breusch-Pagan tests. These same tests were used by Fernandes *et al.* (2014), Muniz, Nascimento and Fernandes (2017), and Silva *et al.* (2021) to check the assumptions of the residuals obtained from fitting non-linear models. Table 1 lists the results of these tests for the three models for the dry mass of the stems.

Table 1. P-value of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests, applied to the residuals of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn stems

Previous crop	Model	SW	BP	DW
B. brizantha	Logistic	0.300	0.018*	0.346
	Gompertz	0.331	0.015*	0.757
	von Bertalanffy	0.056	0.027*	0.664
Millet	Logistic	0.348	0.311	0.465
	Gompertz	0.317	0.237	0.610
	von Bertalanffy	0.518	0.223	0.613
Common bean	Logistic	0.981	0.381	0.148
	Gompertz	0.800	0.324	0.263
	von Bertalanffy	0.548	0.309	0.315

*significant at 5% probability.

Observing the results in Table 1, with the Shapiro-Wilk test, the assumption of normality of residuals was met in all models and all previous crops (p-value >0.05).

The assumption of homogeneity of variance of residuals was violated (p-value < 0.05) by the Breusch-Pagan test in all models in the previous crop of *Brachiaria brizantha*, showing the heterogeneity of variances. Therefore, the parameters were estimated using the weighted least squares method, so the points with greater variability influence less the parameter estimates. The weighting factor was estimated using the “weights” argument, in relation to the tested classes, "varIdent()" was the best fit for the three models. Heteroscedasticity of variances between measurements is common when models are fit to data obtained over time, as plants develop and there is greater variation as they grow (Fernandes *et al.*, 2014). As the data are cross-sectional, there is heteroscedasticity of variances, as the measurement is carried out in different plants, with variability between them.

And the Durbin-Watson test indicated the assumption of independence of residuals was met in all cases (p-value > 0.05), showing that the residuals were not autocorrelated. As expected in data analysis whose collection was carried out by the cross-sectional method since the measurements were not taken on the same plants. The same was reported by Ribeiro *et al.* (2018), in which horses from different breeders were observed, characterizing a cross-sectional growth study.

Indices of the goodness of fit of the models for the dry mass of stems are listed in Table 2. From the results obtained, all the models fit well with the data. However, the Logistic model showed lower intrinsic (c^J) and parametric (c^θ) non-linearity values. According to Zeviani (2012), a model should be selected over another if it presents the lowest values of both intrinsic and parametric non-linearity. In this case, the Logistic model is the most suitable for all previous crops.

Table 2. Indices of the goodness of fit of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn stems

Previous crop	Model	R^2	RSD	AIC	c^J	c^θ
B. brizantha	Logistic	0.989	20.084	128.352	0.165	0.298
	Gompertz	0.991	18.504	126.058	0.182	0.325
	von Bertalanffy	0.990	4.659	126.513	0.604	0.351
Millet	Logistic	0.958	45.360	151.165	0.285	0.572
	Gompertz	0.963	43.710	150.127	0.326	0.699
	von Bertalanffy	0.964	44.060	150.351	0.544	0.844
Common bean	Logistic	0.918	77.950	166.325	0.470	0.813
	Gompertz	0.938	70.430	163.483	0.506	0.834
	von Bertalanffy	0.945	67.510	162.297	1.097	0.815

Table 3 presents the interval estimates at the 95% confidence level of the parameters of the Logistic models for the dry mass of stems.

Table 3. 95% confidence intervals for the parameters of the models that best fit the data for the dry mass of stems (gram/m^2) in corn plants grown in an area with previous crops of *Brachiaria brizantha*, millet, and common bean

Previous crop	Model	Parameter	Estimate	LI	LS
B. brizantha	Logistic	α	504.641	487.903	521.379
		β	41.257	39.652	42.862
		κ	0.150	0.118	0.182
Millet	Logistic	α	586.047	546.260	630.534
		β	41.722	38.128	45.572
		κ	0.121	0.081	0.188
Common bean	Logistic	α	721.947	659.176	794.138
		β	37.954	33.836	43.055
		κ	0.168	0.087	0.378

The parameters in Table 3 were all significant at the 5% probability level, as their confidence intervals did not contain zero, indicating that the Logistic model was appropriate to describe the accumulation of the dry mass of stems in relation to the days after the emergence of plants. According to the interval estimates for the Logistic model that best fit the data in the three straw mulches, in the common bean straw, there was a greater accumulation of the dry mass of the stems (659.176 g/m^2 to 794.138 g/m^2).

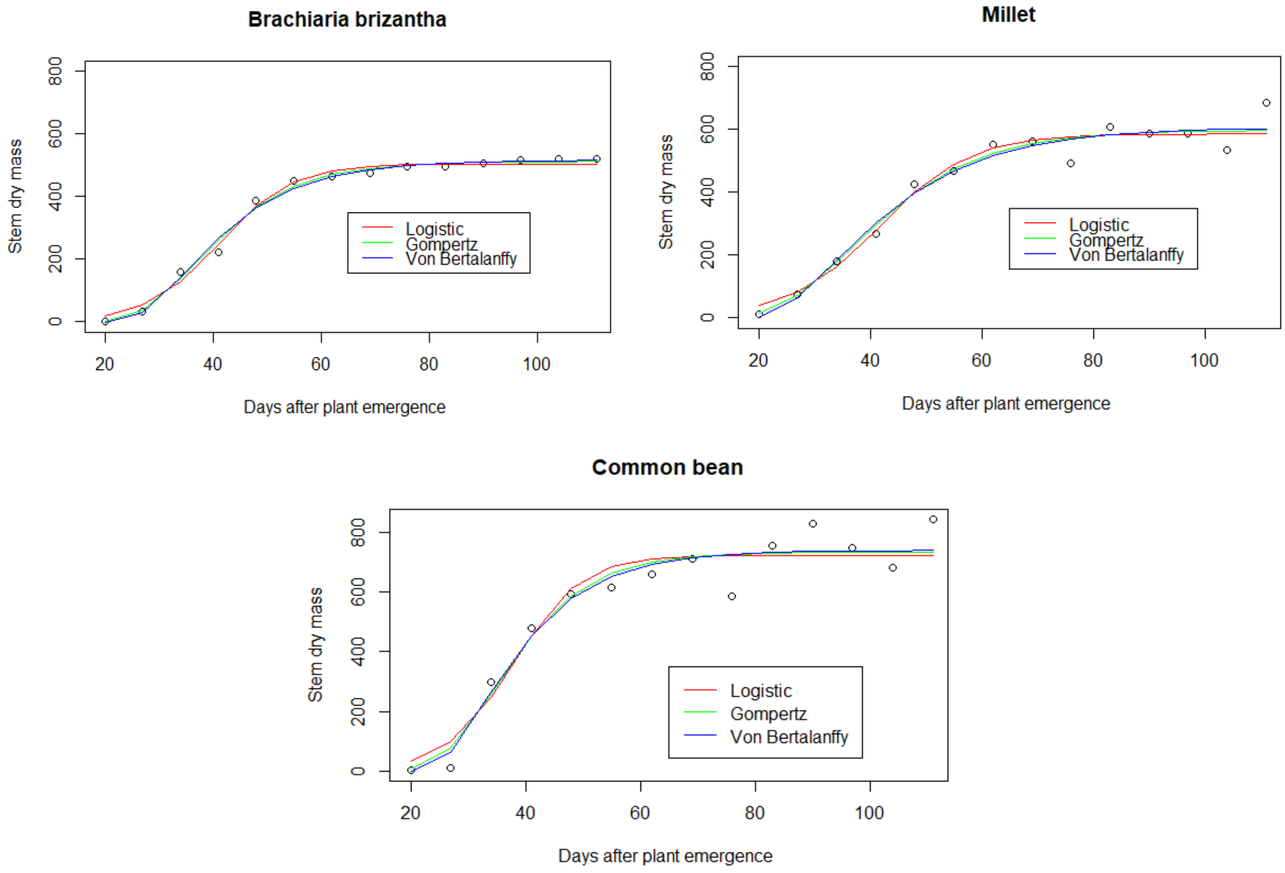
The results of the α parameter found are similar to those of Oliveira *et al.* (2013) (700 g/m^2, 560 g/m^{2,} and 500 g/m^2, for common bean, millet, and *B. brizantha* straw mulches, respectively), which indicate that the highest accumulation of the dry mass of stems occurred at the end of the cycle.

The fit of the models for the dry mass of stems is illustrated in Figure 1, which shows that the Logistic model fitted well with the data. Based on the punctual estimates of the parameters, the inflection points (IP), the point of maximum acceleration (PMA), and the point of maximum deceleration (PMD) of the model were found for each previous crop.

For the dry mass of stems of corn plants cultivated in an area previously grown with a crop of *B. brizantha*, based on the Logistic model, the inflection point (IP) occurred approximately 41 days after plant emergence. At this point, the estimate of the ordinate was 252.320 g/m^2. The point of maximum acceleration (PMA) occurred 33 days after plant emergence, with a maximum accumulation of 106.643 g/m^2, and the point of maximum deceleration (PMD) occurred 50 days after plant emergence, with a maximum accumulation of 397.997 g/m^2.

For the dry mass of stems of corn plants cultivated in an area previously grown with Millet, based on the Logistic model, the IP also occurred approximately 42 days after plant emergence. At this point, the ordinate estimate was 293.023 g/m^2. PMA occurred 31 days after plant emergence, with a maximum accumulation of 123.846 g/m^2, and PMD occurred 53 days after plant emergence, with a maximum accumulation of 462.200 g/m^2.

And for the dry mass of stems of corn plants cultivated in an area previously grown with a crop of Common Bean, based on the Logistic model, the IP occurred approximately 38 days after plant emergence, in which the estimated ordinate was 360.9703 g/m^2. PMA occurred 30 days after plant emergence, with a maximum accumulation of 152.565 g/m^2, and PMD occurred 46 days after plant emergence, with a maximum accumulation of 569.381 g/m^2.



Brachiaria brizantha

Millet

Common bean

Figure 1. Fit of the Logistic, Gompertz, and von Bertalanffy models for the accumulation of the dry mass of stems (gram/m^2) according to different previous crops.

3.2 Dry mass of leaves

According to the results in Table 4 for fitting the proposed models for the dry mass of corn leaves, the assumption of normality was not met in the Logistic model for the previous crop of *Brachiaria brizantha*. In this case, we decided to work with models that did not violate this assumption, as also performed by Silva *et al.* (2020). The non-normality of residuals was also observed by Lima *et al.* (2017) in the study of non-linear models for the description of boron accumulation in different parts of the cultivar Jalo bean, and also by Mangueira *et al.* (2016), who used the logistic model considering different distributions for the errors applied to corn plant height data.

The assumption of homogeneity of variances was also not met in the Logistic and Gompertz models with the previous crop of *Brachiaria brizantha*. Since the weighting factor is estimated using the “weights” argument, in relation to the tested classes, "varIdent()" was a better fit for the Logistic and Gompertz models. The assumption of independence of residuals was met in all models and in all previous crops, which is common in cross-sectional data collection.

Table 4. P-value of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests, applied to the residuals of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn leaves

Previous crop	Model	SW	BP	DW
B. brizantha	Logistic	0.022*	0.004*	0.973
	Gompertz	0.682	0.004*	0.929
	von Bertalanffy	0.709	0.188	0.583
Millet	Logistic	0.051	0.441	0.476
	Gompertz	0.120	0.210	0.229
	von Bertalanffy	0.183	0.051	0.150
Common bean	Logistic	0.890	0.386	0.061
	Gompertz	0.943	0.400	0.101
	von Bertalanffy	0.897	0.567	0.110

*significant at 5% probability.

Based on the indices of the goodness of fit of the models (Table 5), the values were close in all the criteria, however, the Gompertz model presented a higher R^2 and lower values of RSD, AIC, and Bates and Watts curvatures, in all previous crops, proving to be more adequate to describe the accumulation of the dry mass of leaves. Moura *et al.* (2008) show that the Gompertz model presented the best fit for simulating the growth of cowpea and corn plants.

Table 5. Indices of the goodness of fit of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn leaves

Previous crop	Model	R^2	RSD	AIC	c^J	c^θ
B. brizantha	Gompertz	0.991	8.540	104.409	0.173	0.274
	von Bertalanffy	0.990	9.672	107.892	0.405	0.375
Millet	Logistic	0.989	9.635	107.784	0.187	0.317
	Gompertz	0.992	8.742	105.062	0.151	0.251
	von Bertalanffy	0.991	9.553	107.547	1.135	0.866
Common bean	Logistic	0.933	34.600	143.583	0.562	1.136
	Gompertz	0.937	34.180	143.238	0.384	0.788
	von Bertalanffy	0.933	34.787	143.733	2.276	2.637

Table 6 lists the estimates of the parameters of the Gompertz model, which best fit the data, with 95% confidence intervals for the dry mass of leaves. In the confidence interval of the common bean previous crop, there was a greater accumulation of leaf dry mass (327.450 g/m^2 to 377.553 g/m^2).

The interval estimates of α had close values, similar to those found by Oliveira *et al.* (2013) with 350 g/m^2, 300 g/m^2, and 270 g/m^2, for common bean, millet, and *B. brizantha* straw mulches, respectively, verifying that the highest accumulation of the dry mass of leaves occurs at flowering because the plant prioritizes the production of leaves for photoassimilate production.

Table 6. 95% confidence intervals for the parameters of the models that best fit the data for the dry mass of leaves (gram/m^2) in corn plants grown in an area with previous crops of *Brachiaria brizantha*, millet, and common bean

Previous crop	Model	Parameter	Estimate	LI	LS
B. brizantha	Gompertz	α	269.661	262.772	276.550
		β	30.003	28.841	31.164
		κ	0.118	0.097	0.138
Millet	Gompertz	α	271.747	265.386	278.194
		β	30.329	29.390	31.247
		κ	0.183	0.149	0.225
Common bean	Gompertz	α	351.739	327.450	377.553
		β	30.553	27.750	34.192
		κ	0.211	0.105	1.060

The fit of the models for the leaf dry mass is shown in Figure 2, which indicates that the Gompertz model fitted well with the data. Based on the punctual estimates for the Gompertz model in the three previous crops, we observed the following critical points:

For the dry mass of leaves of corn plants grown in an area previously cultivated with a crop of *B. brizantha*, the inflection point occurred approximately 30 days after plant emergence, where the ordinate estimate was 99.213 g/m^2. The point of maximum acceleration (PMA) occurred 22 days after plant emergence, with a maximum accumulation of 32.432 g/m^2, and the point of maximum deceleration (PMD) occurred 38 days after plant emergence, with a maximum accumulation of 239.646 g/m^2.

For the dry mass of leaves of corn plants grown in an area with a previous crop of **Millet**, the inflection point also occurred approximately 30 days after plant emergence. At this point, the ordinate estimate was 99.980 g/m^2. The point of maximum acceleration (PMA) occurred 25 days after plant emergence, with a maximum accumulation of 32.683 g/m^2, and the point of maximum deceleration (PMD) occurred 36 days after plant emergence, with a maximum accumulation of 241.500 g/m^2.

For the dry mass of leaves of corn plants cultivated in an area with a previous crop of Common Bean, the inflection point occurred 30 days after plant emergence, when it reached a maximum of 129.410 g/m^2. The point of maximum acceleration (PMA) occurred 26 days after plant emergence, with a maximum accumulation of 42.320 g/m^2, and the maximum deceleration point (PMD) occurred 35 days after plant emergence, with a maximum accumulation of 312.589 g/m^2.

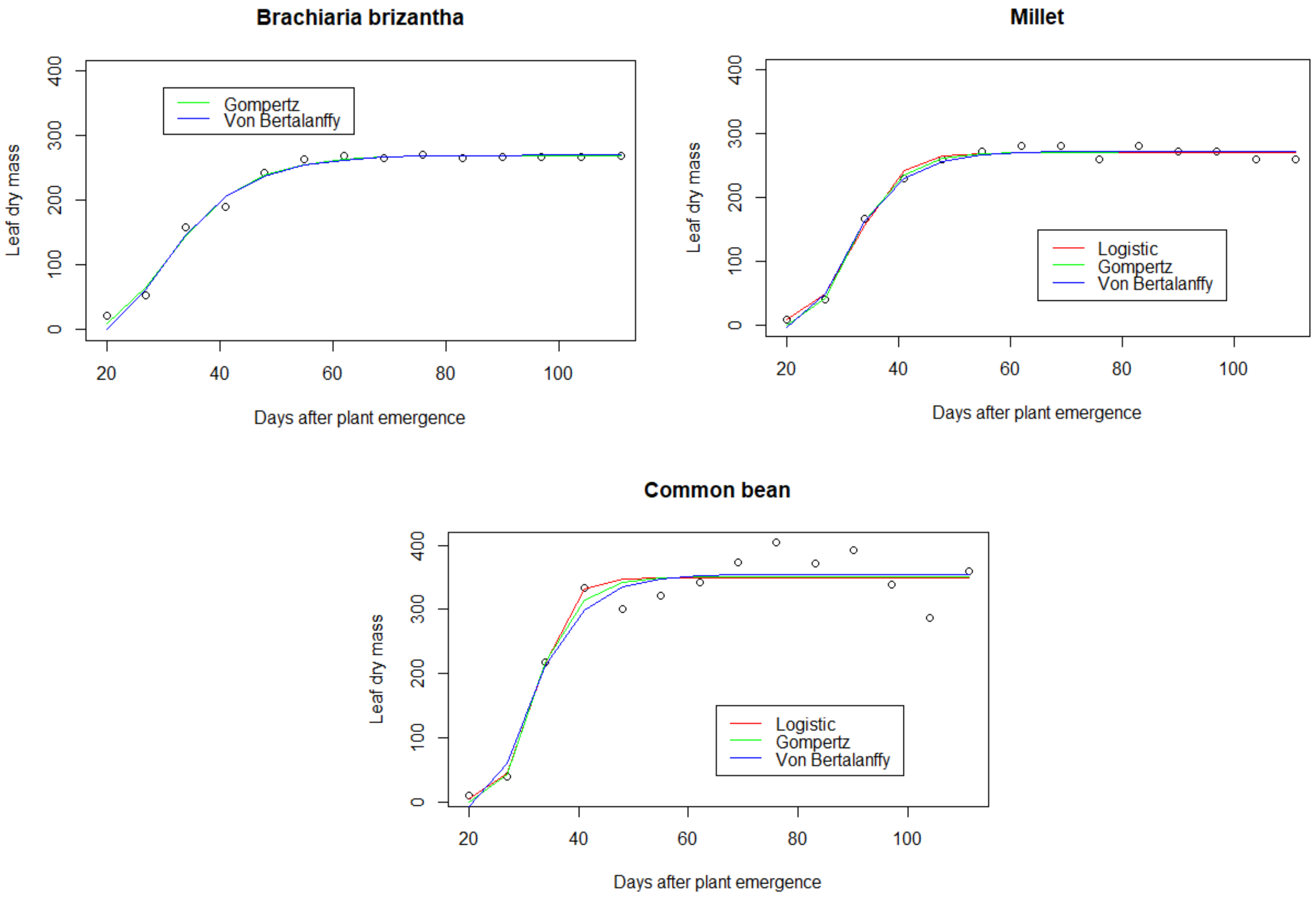


Figure 2. Fit of the Logistic, Gompertz, and von Bertalanffy models for the accumulation of the dry mass of leaves (gram/m^2) according to different previous crops.

3.3 Dry mass of ears

Table 7 lists the results obtained in the application of the Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson tests of the three models for the dry mass of corn ears.

According to the Shapiro-Wilk test, the assumption of residual normality was not met in the von Bertalanffy model in the previous crop of millet (p-value < 0.05). Therefore, we decided to work with models that did not present this violation.

Table 7. P-value of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests, applied to the residuals of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn ears

Previous crop	Model	SW	BP	DW
B. brizantha	Logistic	0.475	0.099	0.031*
	Gompertz	0.413	0.099	0.081
	von Bertalanffy	0.909	0.135	0.080
Millet	Logistic	0.219	0.042*	0.272
	Gompertz	0.470	0.086	0.743
	von Bertalanffy	0.024*	0.601	0.577
Common bean	Logistic	0.215	0.012*	0.147
	Gompertz	0.348	0.018*	0.178
	von Bertalanffy	0.746	0.297	0.064

*significant at 5% probability.

The Breusch-Pagan test indicated that the assumption of homogeneity of variances was violated in the Logistic model in the previous crops of millet and common bean, as well as in the Gompertz model in the previous crop of common bean (p-value < 0.05). The weighting factor was estimated using the “weights” argument, and the "varPower()" tested class was the best fit for both models. By estimating the parameters using the weighted least squares method, there was an improvement in the results of the goodness-of-fit indices, with a decrease in the values of AIC and RSD criteria, which was also presented by Fernandes *et al.* (2014).

With the Durbin-Watson test in the previous crop of *Brachiaria brizantha*, there was a violation of the assumption of independence of residuals in the Logistic model (p-value < 0.05), although it was not expected due to the data collection methodology. With this violation, new fits were made and, modeling the dependence of residuals, the parameters were estimated using the generalized least squares method, including the first-order autoregressive term (AR1). Autocorrelated residuals were also verified by Frühauf *et al.* (2020), Frühauf *et al.* (2021), and Frühauf *et al.* (2022) in the study of diametric growth of cedar, beans, and eucalyptus.

The indices of the goodness of fit of the models for the dry mass of corn ears are presented in Table 8. All the models fitted well with the data, and the Logistic model showed lower values of the Bates and Watts curvatures, in all previous crops. Muniz, Nascimento and Fernandes (2017) analyzed the growth of cocoa fruit, in which the Logistic model was also the most efficient in describing growth.

Table 8. Indices of the goodness of fit of the Logistic, Gompertz, and von Bertalanffy models for Dry Mass (DM) of corn ears

Previous crop	Model	R^2	RSD	AIC	c^J	c^θ
B. brizantha	Logistic	0.973	84.490	169.326	0.245	1.304
	Gompertz	0.982	71.660	163.967	0.295	2.577
	von Bertalanffy	0.983	71.100	163.748	1.325	3.249
Millet	Logistic	0.977	2.367	158.025	0.190	0.839
	Gompertz	0.990	59.330	158.682	0.210	1.440
Common bean	Logistic	0.968	2.046	164.229	0.324	1.057
	Gompertz	0.971	34.781	172.593	0.390	2.114
	von Bertalanffy	0.965	151.053	184.847	1.092	2.965

Table 9 lists the estimates of the parameters of the Logistic model, with 95% confidence intervals for the dry mass of corn ears. Considering the heterogeneous variance and adding the first-order autoregressive parameter AR(1) when necessary.

Table 9. 95% confidence intervals for the parameters of the models that best fit the data for the dry mass of ears (gram/m^2) in corn plants grown in an area with previous crops of *Brachiaria brizantha*, millet, and common bean

Previous crop	Model	Parameter	Estimate	LI	LS
B. brizantha	Logistic	α	1459.768	1086.922	1832.612
		β	80.269	69.930	90.609
		κ	0.071	0.039	0.103
		φ	0.303	-0.228	0.695
Millet	Logistic	α	1354.806	1151.200	1558.412
		β	70.655	65.827	75.482
		κ	0.113	0.098	0.137
Common bean	Logistic	α	1873.257	1537.291	2209.223
		β	74.771	69.137	80.405
		κ	0.103	0.083	0.124

Observing the results in Table 9, the limits of confidence intervals did not include zero, except the φ parameter in the previous crop of *Brachiaria brizantha*. Although the test of independence of residuals for the logistic model indicated first-order autocorrelation and the model was fit by generalized least squares methods, the confidence interval of the autoregressive parameter φ included zero, demonstrating the non-significance of this parameter. There was an intersection between the three intervals, indicating that the three previous crops have common values for the accumulation of the dry mass of ears. Regarding the results of the α and β parameters found for the Logistic model, they are similar to those of Oliveira *et al.* (2013), in which the ears showed increasing values of dry matter from their emergence until, practically, the end of the cycle, representing, among the evaluated structures, the highest accumulation of dry mass. For Souza, Luís and Piletti (2016) what determines the productivity potential are characteristics such as ear length, diameter, number of ears per area, and grain density.

Figure 3 presents the fit of the models in each treatment, in which the Logistic model presented a better fit to the data in the straw mulches of *B. brizantha*, millet, and common bean. Finding the critical points for the Logistic model in each treatment, we have:

For the dry mass of ears of corn plants cultivated in an area previously grown with *Brachiaria brizantha*, with the Logistic model, the inflection point occurred approximately 80 days after plant emergence, when it reached a dry mass of 729.884 g/m^2. The point of maximum acceleration (PMA) occurred 62 days after plant emergence, with a maximum accumulation of 308.485 g/m^2, and the point of maximum deceleration (PMD) occurred 98 days after plant emergence, with a maximum accumulation of 1,151.282 g/m^2.

For the dry mass of ears of corn plants grown in an area previously planted with **Millet**, based on the Logistic model, the inflection point occurred approximately 71 days after plant emergence. In this case, the estimated ordinate was 677.403g/m^2. The point of maximum acceleration (PMA) occurred 59 days after plant emergence, with a maximum accumulation of 286.304 g/m^2, and the point of maximum deceleration (PMD) occurred 83 days after plant emergence, with a maximum accumulation of 1,068.502 g/m^2.

And for the dry mass of ears of corn plants cultivated in an area with a previous crop of Common Bean, also based on the Logistic model, the inflection point occurred approximately 75 days, when the estimate of the ordinate was 936.628 g/m^2. The point of maximum acceleration (PMA) occurred 62 days after plant emergence, with a maximum accumulation of 395.865 g/m^2, and the point of maximum deceleration (PMD) occurred 87 days after plant emergence, with a maximum accumulation of 1,477.391 g/m^2.

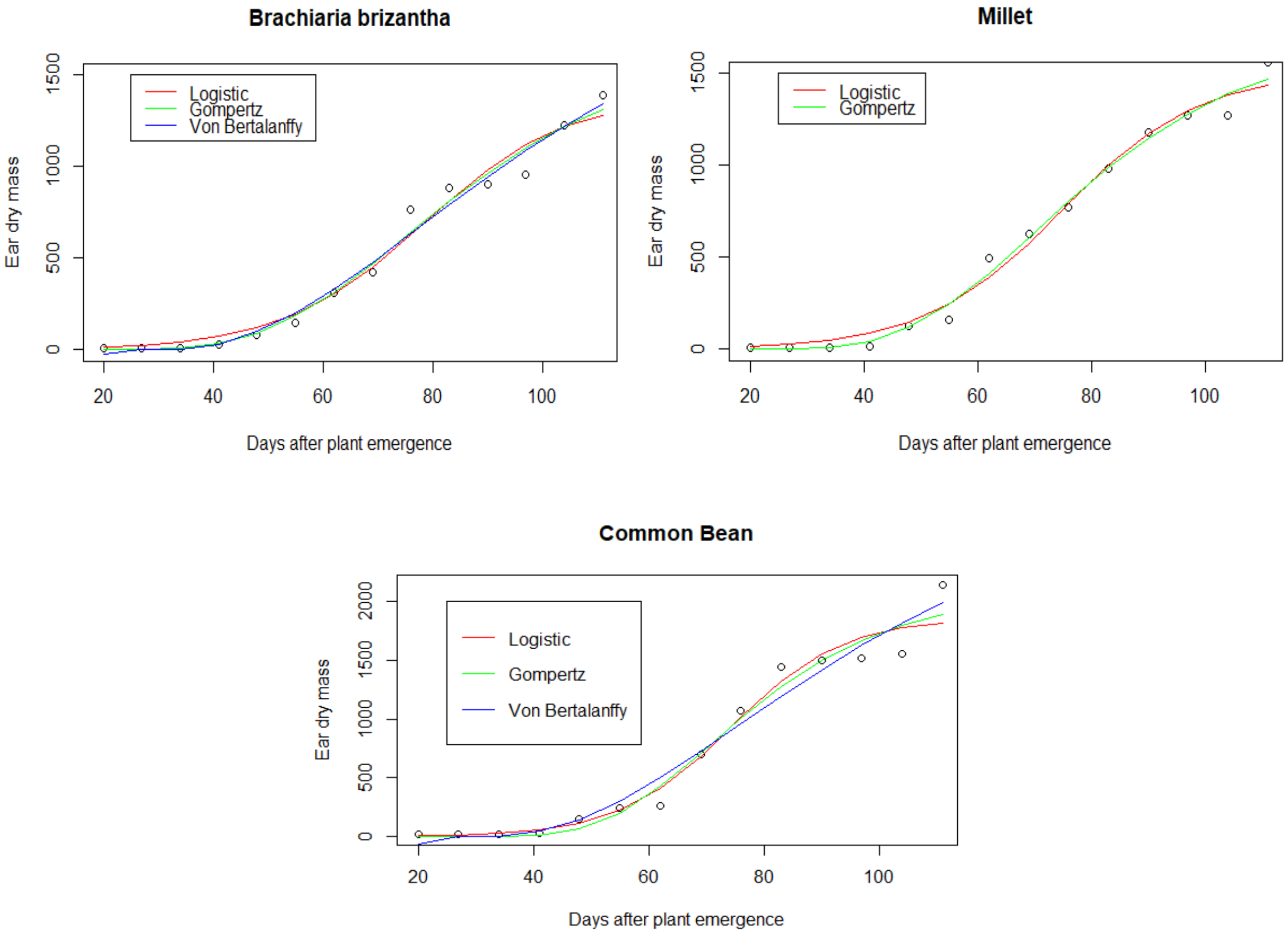


Figure 3. Fit of the Logistic, Gompertz, and von Bertalanffy models for the accumulation of the dry mass of ears (gram/m^2) according to different previous crops.

3.4 Total dry mass

Table 10 lists the results obtained in the application of the Shapiro-Wilk, Durbin-Watson, and Breusch-Pagan tests for the three models (Logistic, Gompertz, and von Bertalanffy) in the total dry mass (considering stems, leaves, and ears) for the previous crops of *Brachiaria brizantha*, millet, and common bean.

Table 10. P-value of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests, applied to the residuals of the Logistic, Gompertz, and von Bertalanffy models for the total dry mass of corn plants

Previous crop	Model	SW	BP	DW
B. brizantha	Logistic	0.796	0.074	0.008*
	Gompertz	0.962	0.072	0.067
	von Bertalanffy	0.975	0.070	0.131
Millet	Logistic	0.189	0.279	0.222
	Gompertz	0.870	0.228	0.695
	von Bertalanffy	0.808	0.209	0.840
Common bean	Logistic	0.821	0.013*	0.078
	Gompertz	0.576	0.037*	0.117
	von Bertalanffy	0.769	0.051	0.126

*significant at 5% probability.

With the results in Table 10, and considering a significance level of 5%, the assumption of normality was met in all models and all straw mulches, that is, the p-value of the Shapiro- Wilk test was greater than 0.05.

The Breusch-Pagan test indicated that the assumption of homogeneity of variance of residuals in the Logistic and Gompertz models for the previous crop of common bean was not met (p-value < 0.05), showing the heterogeneity of variance in residuals from these models. The weighting factor was estimated using the “weights” argument, in relation to the tested classes, "VarPower()" was the best fit for the models. Carvalho *et al.* (2014) fitted non-linear fixed effects, weighted, and mixed-application models, and concluded that weighted models corrected the heteroscedasticity.

In turn, the Durbin-Watson test evidenced that, for the Logistic model in the previous crop of *Brachiaria brizantha*, the residuals presented autocorrelation (p-value<0.05). With this violation, the parameters were estimated using the generalized least squares method, including the first-order autoregressive term (AR1). Autocorrelated residuals were also found by Muianga *et al.* (2016) in the description of the growth curve of cashew fruits by non-linear models and by Silva *et al.* (2021) in the study of the growth curve of Eucalyptus Grandis x Eucalyptus Urofila in different location classifications.

Table 11 lists the results of the indices of the goodness of fit of the three models for the total dry mass of corn plants in the previous crops of *B. brizantha*, millet, and common bean.

Table 11. Indices of the goodness of fit of the Logistic, Gompertz, and von Bertalanffy models for the total dry mass of corn plants

Previous crop	Model	R^2	RSD	AIC	c^J	c^θ
B. brizantha	Logistic	0.973	124.738	179.657	0.164	1.971
	Gompertz	0.983	98.030	172.742	0.142	1.494
	von Bertalanffy	0.986	90.530	170.514	0.168	2.122
Millet	Logistic	0.974	133.600	181.409	0.158	0.934
	Gompertz	0.983	111.600	176.373	0.138	0.859
	von Bertalanffy	0.985	105.700	174.842	0.160	2.092
Common bean	Logistic	0.960	67.411	196.730	0.202	1.246
	Gompertz	0.964	7.683	192.573	0.197	1.235
	von Bertalanffy	0.967	202.500	193.055	0.233	3.568

For the total dry mass in the previous crops of *Brachiaria brizantha*, millet, and common bean, the Gompertz model was more adequate because it presents lower values of Bates and Watts curvatures, and intrinsic and parametric values in all previous crops.

Table 12. 95% confidence intervals for the parameters of the models that best fit the data for total (leaves, stems, ears) dry mass in gram/m^2 in corn plants grown in an area with previous crops of *Brachiaria brizantha*, millet, and common bean

Previous crop	Model	Parameter	Estimate	LI	LS
B. brizantha	Gompertz	α	2430.000	2089.459	3121.420
		β	55.330	49.434	66.391
		κ	0.034	0.023	0.046
Millet	Gompertz	α	2719.000	2351.287	3443.302
		β	54.080	48.364	64.437
		κ	0.034	0.023	0.046
Common bean	Gompertz	α	3253.959	2561.098	3946.819
		β	52.675	44.397	60.953
		κ	0.039	0.026	0.052

In Table 12, the limits of confidence intervals did not include zero. Analyzing the interval estimates of the Gompertz model in each previous crop, there was an intersection between the three intervals, indicating that the three previous crops have common values of accumulation of total dry mass. The results of the α parameter found are similar to those of Oliveira *et al.* (2013), noting a greater accumulation of dry matter by corn plants grown on common bean straw mulch (3,000 g/m^2), followed by pearl millet (2,250 g/m^2) and, finally, *B. brizantha* (2,000 g/m^2). Carvalho, Bianco and Bianco (2013) and Carvalho *et al.* (2007) analyzed the accumulation of total dry mass in corn plants using a second-degree polynomial model, in which the maximum accumulation of dry mass was 150 g/plant, occurring 120 days after plant emergence, at the end of the experimental period.

Evaluating the effect of straw mulches on productivity, the highest values of dry mass accumulation were observed for common bean straw. This suggests that common bean straw provided better conditions for corn development, showing positive effects on productivity, conditioned to the fact that common bean releases nutrients faster, such as nitrogen, which is so important for corn development, corroborating the results presented by Oliveira *et al.* (2013).

Figure 4 illustrates the fit of the Logistic, Gompertz, and von Bertalanffy models

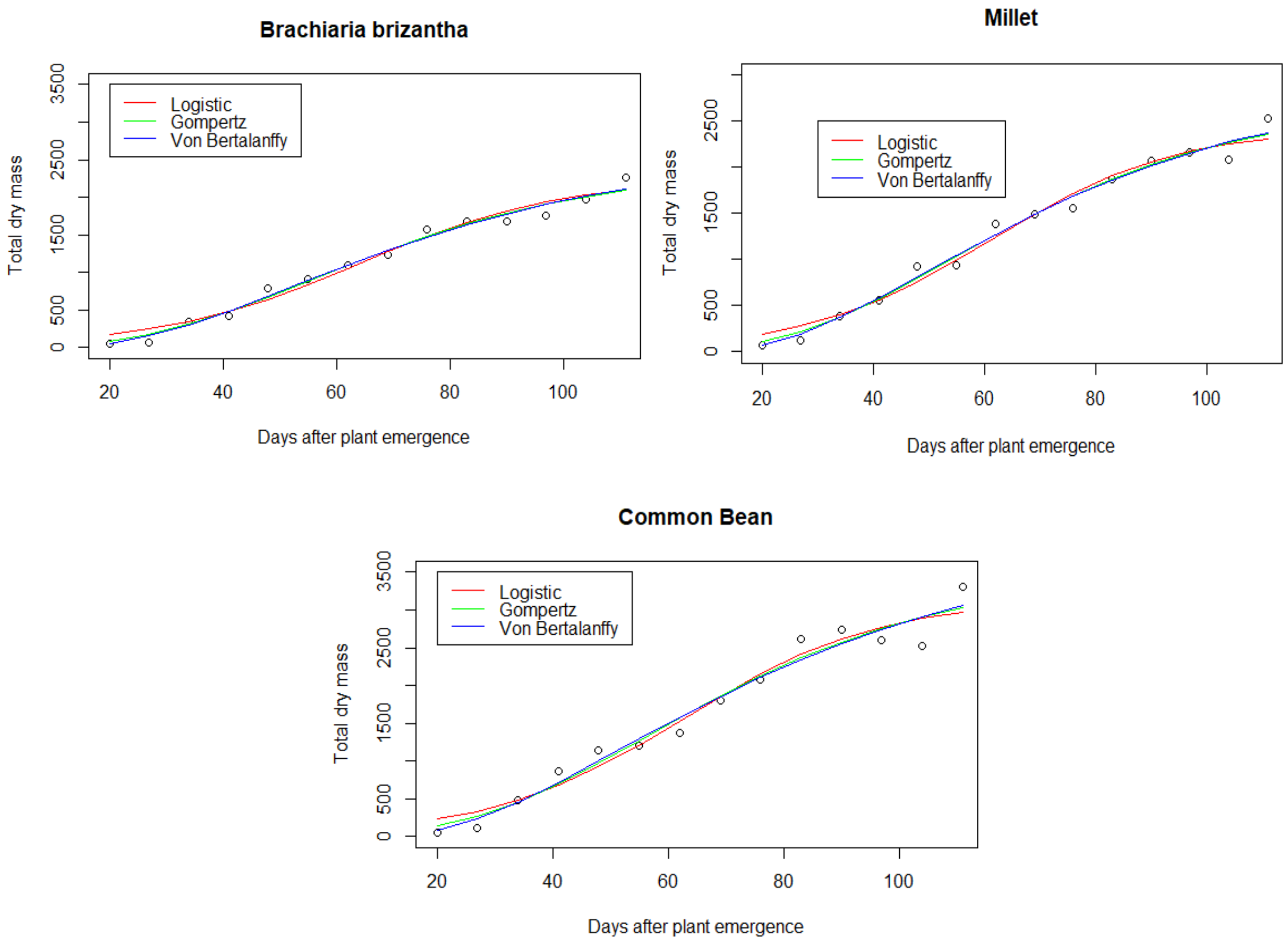


Figure 4. Fit of the Logistic, Gompertz, and von Bertalanffy models for the accumulation of total dry mass according to different previous crops.

The three evaluated models fit well with data on the total dry matter of corn plants in each previous crop. And finding the critical points of the Gompertz model for each predecessor crop, we have:

For the total dry mass of corn plants grown in an area previously planted with *Brachiaria brizantha*, the IP occurred approximately 55 days after emergence, and the ordinate estimate was 894.039 g/m^2. The point of maximum acceleration (PMA) occurred 27 days after plant emergence, with a maximum accumulation of 292,260 g/m^2, and the point of maximum deceleration (PMD) occurred 84 days after plant emergence, with a maximum accumulation of 2,159.531 g/m^2.

For the total dry mass of corn plants cultivated in an area with a previous crop of Millet, the IP in the Gompertz model, the estimated ordinate was 1,000.368 g/m^2, approximately 54 days after plant emergence. The point of maximum acceleration (PMA) occurred 26 days after plant emergence, with a maximum accumulation of 327.019 g/m^2, and the point of maximum deceleration (PMD) occurred 82 days after plant emergence, with a maximum accumulation of 2,416.365 g/m^2.

And for the total dry mass of corn plants cultivated in an area previously grown with a crop of Common Bean, based on the Gompertz model, the estimated ordinate of the PI was 1,197.188 g/m^2, approximately 53 days after plant emergence. PMA occurred 28 days after plant emergence, with a maximum accumulation of 391.359 g/m^2, and PMD occurred 78 days after plant emergence, with a maximum accumulation of 2,891.781 g/m^2.

4. Conclusions

To describe the maximum accumulation of stem dry mass in corn plants grown in an area previously cultivated with a crop of *Brachiaria brizantha*, millet, or common bean, the most appropriate model was the Logistic model.

For leaf dry mass accumulation of corn plants cultivated in an area with a previous crop of *Brachiaria brizantha*, millet, or common bean, the Gompertz model was more appropriate.

In the accumulation of the dry mass of corn ears, the Logistic model fitted better to the data in corn plants cultivated in an area with a previous crop of *Brachiaria brizantha*, millet, or common bean.

As for the maximum accumulation of total dry mass in corn plants cultivated in an area previously planted with a crop of *Brachiaria brizantha* and millet, the Gompertz model fitted better to the data in the three previous crops.

The highest values of dry mass accumulation in stems and leaves were obtained when corn plants were cultivated on common bean straw mulch. For the dry mass of the ears and total dry mass, all the straw mulches resulted in the same accumulation of dry mass.

Acknowledgments

To the Coordination for the Improvement of Higher Education Personnel (CAPES) for the Master's degree scholarship, granted to the first author. The authors thank the two reviewers and the editor for their valuable comments.

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